7ci fgY'Cj Yfj]Yk

Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
DYHYFK\]HY		H\i fgXUm%"%) 'da '! &"%) 'da 'fd'YUgY Ya U]`'hc'UffUb[Y'Ub Uddc]bha YbhŁ	K YVghYf``YjY` 'žFA'' ‰	L)&'*(

GWXcc`7cbHJWh =bZcfa Uh]cb

Fcca " %&ž~Yj Y~" FcVYfhK YVghYf~6i]`X]b[

D\cbY. f\$&£'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##qUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ffg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck``YX[Y'h\Y'HfUX]fJcbU``7i\ghcX]UbgZh\Y'6YXY[U'`fPYbg]b[hcb'WJa\di\głZ';\UX][U'`fP\]hmUbX'5fh\/\'8Yg][b'7Ua\di\gYgL'UbX'h\Y'B[i\bbUk\U'\dYcd'Y'f5i\ghfU']Ub'8YZYbW':\cfW'5WJXYa\mi]b'7UbVYffU\LcZh\Y'`UbXg'k\YfY'YUW\'WJa\di\g'cZI\BGK\']g\cowJhYX''$

5ggYgga Ybh

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5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
Dcfh7c`]c'fj Uf]ci g'a YX]UŁ'	&) ı	Bch5dd`]WUV`Y	%ž&Ž Ž(
A YX]U Webhybhidfydufuhjeb UbX dfcXi Whjeb hUg_'%	' \$ı	Bch5dd`]WUV`Y	%ž&Ž Ž(
A YX]U WcbhYbhdfYdUfUh]cb UbX dfcXi Wh]cb hUg_ &	()।	Bch5dd`]WUV`Y	%ž&Ž ž(

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% DcfhZc`]c fJ Uf]ci g a YX]UŁ

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. '€X]j]Xi U'‰\$\$ k cfXg Yei]j U'Ybh YYXVUW_j]U'@A G

5XX]h]cbU`XYhJ]`g.

GYY'Wci fgY'A ccX`Y'g]hY'Zcf'XYhU]`g

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& A YX]U WcbhYbhdfYdUfUh]cb UbX dfcXi Wh]cb HJg_ '%

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. ⁴bX]j]Xi U`%*\$\$ k cfXg Yei]j U`Ybh YYXVUW_j]U @A G

5XX]h]cbU`XYhJ]`g.

 $\hbox{GYY'W\'ci fgY'A ccX`Y'g]hY'Zcf`XYhJ]`g"}\\$

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . A YX]U WcbhYbhdfYdUfUh]cb UbX dfcXi Wh]cb HJg_ &

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. '±x]j]Xi U'% \$\$ k cfXg Yei]j UYbh YYXVUW_j]U'@A GH\]g]g h\Y Z]bU`UggYgga a Ybh

5XX]h]cbU`XYhJ]`g.

 $\label{eq:constraint} GYY^*Wci fgY^*A ccX^*Y^*g]hY^*Zcf^*XYhJ]^*g''$

 $\label{eq:higher_problem} \mbox{Hi fb]h]b$^{-}gYh]b[.^{-}H\]g$^{-}g$^{-}bchU$^{-}Hi fb]h]b$^{-}Ugg][$ba^{-}Ybh^{-}$

5 https://bxubwww.Fyei]fya Ybhg

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'ha YHUV'Y

H]a YtUV`Y

8 UhY	HmdY	7cbhYbh
K YY_'%' %+': YVfi Ufm!' &% : YVfi Ufm	@YWnifY	%"'+b!XYdh\'X][]hU`'&ifbU`]ga.'W&bWY]j]b[ž fYgYUfW\]b['UbX'dfcXiW]b['diV`]g\UV`Y'df]bHgfmYž UiX]c'UbX'j]XYc'dUW_U[Yg
		D@95G9:5@GC:BCH9:H<5H:HI HCF=5@G:5@GC GH5FH:H<=G:K 99?"
	Hi hcf]U`	BchY'h\Uhhi hcf]U`g'VY[]b'h\]g'k YY_"
		fGYY'AccX`Y'kYVg]hY'Zcf'idXUhYX']bZcfaUh]cbUVcihh\Y'hihcf]U`"Ł
		OH\Y'Zc``ck]b['gWXYXi`Y']g'cb`mdfcj]g]cbU``!'giVYWhhc'WXUb[Y'Ug'h\Y'gYaYghYf'ibZc`Xg""""'WffYbhUg'cZ:YVfiUfmQ
		8if]b['h\Y'Hihcf]U`
		%": +blfcXi Whcbgžcj Yfj]Yk cZWci fgY WcbhYbhž UggYgga Ybhžhi hcf]U``dfYdUfUh]cbg"``
		&"'DfYdUfUh]cb'Zcf'DfcXiWf]cb'DfcYWf'%'DfcZ]`Y fdf]bhghm`Y'UbX'dcXWUghjYfg]cbgŁ
K YY_'&.'&(': YVfi Ufm!'&, : YVfi Ufm	@YWhi fY	; Yth]b['fc'[f]dg'k]t\'dfcZ]`Yg'cZU``'lmdYg'fdf]btz dcXWJgtz'j]XYc'YtWz'!'g\cftYfz'`cb[Yfz'E/5'glmiY
	Hi hcf]U`	fGYY'A ccX`Y'k YVg]hY'Zcf'i dXUhYX']bZcfa Uh]cb UVci h'h\Y'hi hcf]U`'!'gW\YXi `Y'a Um'W\Ub[Y'Ug W'ffYbh'bYk g'Yj Ybhg'i bZc`X"Ł
		%"`8]gW gg]cb`Ufci bX`gUa d`Y#a cXY```dfcZ]`Y ZYUhi fYg`fYI d`cf]b[`X]ZZYfYbhghm`Ygžghfi Whi fYgž'i gY cZbUffUh]j Y`Y`Ya YbhgžXY[fYY`cZgi V^YWh]j]hmYhWŁ
		&"'DfcZ]`Y'dfcXi Wfjcb'HYUa g'gYHiY'cb']bHYfj]Yk gi V'YWY'dfYdUfY'U'g\cfhgYhicZdcgg]V`Y'ei YgHjcbg/ Vf]YZfYdcfhhc'H\Y'WUgg'fk \m]g'H\]g'dYfgcbfij'ghcfm dcHYbHJU``m'''di V`]g\UV`Y''3Ł
K YY_" ."&"A UfVX"!"* A UfVX	@YWi fY	%"'9ZZYWNjY']bhYfj]Yk]b['Zcf'df]bhghmY'UbX'UiX]c dfcZjYg.''\ck'hc'[Yhk\Uhmci'bYYX'Zfca'mcif]bhYfj]Yk'giVYWh

I	I	

K_YY_'%\$.'&\$'5df]`'!'&(5df]`	@Y₩i fY	H65 fl]bU``Ugg][ba YbhX]gW gg]cbž'gi [[Ygh]cbg'YhWŁ
		GYY`Wci fgY`A ccX`Y`g]HY`Zcf`XYHJ]`g"

F Ygci fWYg

DfYgWf]VYX F Ygci fWYg'

Bc'dfYgWf]VYX HYI HVcc_

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwb: bh/f'u'dfcv'ya k \yb'uhh/a dh/b['hc'gi va]hmci f'ugg][ba Ybhh\fci [\ 'Hi fb]h/bž'd'YugY

HY'Yd\cbY'9| h/fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U] 'h\Ya 'cb'Yl h/fbu'h/hgi ddcfh4 i bgk "YXi "Ui "'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ" Image: UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h/b'mci 'a UmUdd'mZcf'Ub'Yl h/bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h/w_Yhbi a VYf'Zfca '9| h/fbu''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'YY] UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Yl h/bg]cb'Udd']WJh/cb" Image: Ya U]''9| h/fbu''Gi ddcfhmci 'k]'`
Ui hca Uh/WJ'mfYW'] Y'U'h/w_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'`'bYYX'hc'gdYW/ZWJ'`mUg_'Zcf'cbY''
Hi fb]h/b'U'gc'dfcj]XYg'i dXUh/g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU`
ghfi Whi fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa Uh]cb'k]h\ci h\UW_bck `YX[Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h\UhYX]hUbX'hc
d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k]h\ci h\Uddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI_BGK_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b['UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``ha Ybu[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\žXfUZh]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fWfg`]b`dfYdUf]b[`U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhhUg_g

5 WUXYa]W±bZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb'fY`UhJb['hc.