

MDIA5028

7 f]h]WU`DYfgdYWhj Yg']b'7 ca a i b]WUhjcb

HYfa 'C bY''# '&\$&\$

7ci fgY'Cj Yfj]Yk

Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUhjcb	D\cbY
DUi ``F mXYf		6mUddc]bha Ybh K YXbYgXUm %*\$\$!%+\$\$	' %%6	D`YUgY'Ya U]`

'Hi hcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
5XUa 'GHJd'Yhcb	U"ghJd`Yhcb4 i bgk "YXi "Ui	-	KYVghYf`&'%L #KYVghYf '%%6	D`YUgY'Ya U]`

GWXcc~7cbHJWh~abZcfa Uhlcb

Fcca " %ž~Yj Y`" FcVYfhK YVghYf~6i]`X]b[

D\cbY. f\$&£'-',) '(,)*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##qUa "Ufhq"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GW.cc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5\Lock\`YX[Y'h\Y'HfUX]tjcbU``7i\gtcX]Ubg\cong h\Y'6YXY[U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`YX][U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'\UW\\`WJa\di\g'\cong \cong \cong m5\Lock\Y'\UbXg'k\YfY'YUW\\`WJa\di\g'\cong \cong \cong\cong \cong \cong \cong \cong \cong \cong \cong \cong \cong \cong$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`cZYfg`U`fYj]Yk `cZ\]ghcf]WU``mg][b]Z]WUbha cj Ya Ybhg`]b`Wf]h]WU``h\YcfmUbX`U]a g`hc`YghUV`]g\Ub`UXj UbWX`WcbWfdh`U``i bXYfghUbX]b[`cZU`gYhcZWcffYgdcbX]b[`dYfgdYWh]j Yg`]b`Wca a i b]WUh]cb" H\fci [\`U`Wcbg]XYfUh]cb`cZWUgg]WU``h\fci [\`dcgha cXYfb`a cXY`g`cZWca a i b]WUh]cbz`mci `k]````YUfb`h\Y j U`i Y`cZh\YcfmUg`Vch\`U`a cXY`cZ]bei]fmUbX`Ug`U`Wfi W]U``d`UhZcfa `Zcf`ghfUhY[]WWca a i b]WUh]cb`XYg][b" 5\W\cfX]b[`mz`mci `k]```YUfb`hc`Udd`ma U'cf`h\YcfYh]WU``ZfUa Yk cf_g`hc`dUfh]W\`Uf`di V`]Wa YX]U`WJa dU][bg"`

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 8]gWrfbžYj U`i UhržUbX`Udd`m'U`bi a VYf`cZWca a i b]WUh]cb`a cXY`g`cf`h\Ycf]Yg`h\Uh`]bZcfa `di V`]W a YX]U`WJa dU][bg"
- &" 5fh]W `UhY`Ub`UXj UbWX`UddfYW]Uh]cb`cZWf]h]WU``dYfgdYWh]j Yg`]b`Wca a i b]WUh]cb`h\fci [\`k\]W\ di V`]Wa YX]U`WUa dU][bq`a][\h'VY`Wf]h]ei YX"'``
- ' " 7 fYUhY 'U'WUa dU][b'UbX'X]gW gg`\ck 'U'fUb[Y'cZWf]h]WU`dYfgdYWf]j Yg']b'Wca a i b]WUh]cb']bZcfa gdYWJZ]WY`Ya Ybhg'cZWUa dU][b'XYg][b'UbX'WUa dU][b'XmbUa]Wg'''

HYUWX]b[GhfUhY[]Yg

 $\label{thm:composition} $$HYY`YWi fYg'dfcj]XY'Ub'cddcfhi b]$$mhc'cZYf'k Y``!XYj Y`cdYX'UbX'bi UbWYX']bg][\hg'Udfcdcg'_YmW'`h fU`a cj Ya Ybhg'UbX'UggcWJUhYX'h\YcfYhJWU``ZfUa Yk cf_gž'k \]`gh'h\Y'hi hcf]U`g'ZUW]`]hUhY']b!XYdh\'X]gWi gg]cb'UbX qhi XYbhYb[U[Ya Ybh'Ufci bX'h\YqY''$

H\Y`Wci fgY`k]``VY`HUi [\hij]U``YWi fYg`UbX`hi hcf]U`g"

5ggYgga Ybh

D`YUgY`gYY`A ccX`Y

5ggYgga YbhHUg_g

5ggYgga Ybh'hUg_	K Y][\h	8i Y [.] 8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
GbUd DfYgYblUljcb	' \$1	Bch5dd`]WUV`Y	%ž&
H\YcfYhJWU`YggUm	(\$1		

C

5 hm/b XUb VW F Yei]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] } \\$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

8UHY	HmdY	7 cbhYbh			
K YY_'% '%+': YVfi Ufm! '&%	(YY_'%'%+': YVfi Ufm!'&%'@YW i fY YVfi Ufm				
: YVfi Ufm					

1	-88 - -81 5@'HA 9H56@9
	D`YUgY`fYZYf`ho`cb`]bY`WcbhYbhidYf`AccX`Yž k\]W\`aighVY`UVWYggYX`UbX`j]YkYX`df]cf`ho`mcif hihof]U```
K YY_'(.'-'A UfW\'!'% A UfW\	Glfi Wi fU`]ga '%

K YY_`+." \$`A UfWX`!" 5df]`	FYUX]b[H< € '€ 'A 8 5 ') \$&, fg 'F 9 5 8 ₺; 'K 9 9 ? "`'H < 9 F 9 € '@A @9 7 HI F 9 "'H < 9 F 9 '5 F 9 '@A 'HI HC F 5 @G
K_YY_', .'*'5df]`'!'%\$'5df]`	@YWn fY	Ghfi Whi fU`]ga '(
		J]gi U^hYl hg
		BC'HI HCF =5@
		DFC>97H'7CBGI@HG'D9F'H+A9H56@9'H<5H K=@@<5J9'699B'89J9@CD98
		; ccX': f]XUmf5df]`'%\$E'XcYg'bch']a dUWi`YWi fY'cf Wcbgi `hg"
K YY_''% '5df]`'!'%+ 5df]`	@YWn fY	DcgHghfi Wi fU`]ga 'UbX'Wci fgY'cj Yfj]Yk
		8 Yff]XU
		8 Y A Ub
		@UWJb^fU[U]bŁ
		? f]gHYj U
		8 Y`Yi nY
		BchYʻUVci hi: ci WUi `higʻYa Yf[YbWYʻUgʻUʻdcghi ghfi WhifU`]gh#[YbYUʻc[]WU`a Yh\cX
		BC'HI HCF-5@
		DFC>97H'7CBGI@HG'D9F'H→A9H56@9'H<5H K@@'<5J9'699B'89J9@CD98
		9UghYf'A cbXUmf6df]`'% E'XcYg'bch']a dUWh`YWh fY'cf Wcbgi `hg"'
K YY_"%\$."&\$"5df]`"!"&(5df]`	±bhY bg]j Y	H\Y``YWnifY`h\]g`kYY_`]g`fYd`UWYX`Vm`gdYW]U``dfc^YWn Wcbgi`hg`f]ZfYei]fYXŁ"`
		BC'HIHCF-5@
		DFC>97H:7CBGI@HG:D9F:H=A9H56@9:HC:69 89J9@CD98

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

 $< Uk_Yg\check{z}'H'f\&\$\$' \; \dot{z}'G'lfi \; WhifU`]ga \; `UbX'GYa \;]ch]Wg'f\&bX'YXb\dot{z}'F \; ci \; hYX[\; Yallow \; fallow \; fallow$

FYWca a YbXYX FYgci fWYg

7\UbX`Yfž`8"`f&\$%+Ł`GYa]ch]Wg.`H\Y`6Ug]Wg`flfX"`YXb"Ł`Fcih`YX[Y

9U[`Yhcbž'H"'f&\$\$, Ł'@]hYfUfm'H\Ycfm'5b'\+blfcXi Wh]cb'fl fX"'YXb"Ł'6`UW_k Y``

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU''Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\'Hi fb]r]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'r]\W_Yhbi a VYf'\Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfg\L'rc
]bWi XY'Ug'Yj]XYbW' 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']\WJh]cb'''=\maintention
UfY'' bUYY''mfYWY']j Y'U'r]\W_Yhbi a VYf\Zfvi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'rc'gdYW]\ZfwJ''mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUh]j Y'gi Va]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU`
ghfi Whi fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa Uh]cb'k]h\ci h\UW_bck `YX[Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h\UhYX]hUbX'hc
d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k]h\ci h\Uddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI_BGK_`cf`Ubch\Yf`i b]j\Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b['UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``hja Y`a UbU[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`hja Y`a UbU[Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'hja Y`Zcf fYgYUfW\žXfUZh]b[`UbX`dfcdYf`fYZ/fYbW]b[`cZgci fWfg`]b`dfYdUf]b[`U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhiHUg_g • i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK