# 7ci fgY'Cj Yfj ]Yk

## GHJZ:7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
DYHYf'K \]HY	- G	, ,	FcVYfh KYVghYf'!'fa ' ‰	
		a U_Y'Ub Uddc]bha Ybhz		

### GWXcc`7cbHJWhi±bZcfa Uhjcb

GVXcc``cZhY`5fhg`UbX`AYX]U

Fcca " %&ž@Yj Y`" žFcVYfhK YVghYf'6i ]`X]b[ ff) %(Ł

D\cbY.'f\$&E'-',)'(,)\*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'`fPYbg]b[tcb'WJa di g\cong \cong \cong mJb'\cong mb\cong \cong \cong m5W_bck \cong YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'`fPYbg]b[tcb'WJa di g\cong \cong \cong mJb'\cong mJb'\cong \cong m5W_J\cong \cong \c$ 

## 7ci fgY 8YHJ] g

#### 7 fYX]hDc]bhg \* \*

### Gi a a UfmcZh\Y'7ci fgY

H\Y'Wci fgY'k]``'VY'WcbXi WfX'Ug'(!\ci f'gYa]bUfg'Xi f]b['k\]WX'mci 'k]``'XYj Y'cd'X][]fU''ci fbU']ga dfc YWfg'Zcf'dchYbfjU''di V']WJfjcb'cf'VfcUXWJghž'Wc'``UVcfUhjj Y'mgi V!YX]h#Yj]Yk 'WUgga UhYgfiWcbhYbhž'XYg][bk YVdU[Yg'Zcf'h\Y'XY']j YfmcZWcbhYbhž'UbXž]b'WJgYg'k\YfY'k cf\_'\Ug'VYYb'di V']g\YXža UbU[Y'UbX a cb]hcf']hg'i dhU\_Y'cb'gcWJU''a YX]U'''9UWX'gYa]bUf'k]``'YggYbhJU``mVY'U'X][]hU''ci fbU']ga '''WJb]W'''HYUWX]b[k]``'df]a Uf]'mhU\_Y'h\Y'Zcfa 'cZfWŁ'U'WcbhJbi ci g'dfcW/gg'cZgi V!YX]hcf]U'!ghmY'ZYYXVUW\_#[i]XUbWY'Vmh\Yhihcf'fUb']bXi ghfmdfcZ/gg]cbU'Ł']b'h\Y'fc'Y'cZYXJhcf'UbX'f&Ł'h\Y'Wc'`UVcfUhJj Y'fYj]Yk]b['Vmghi XYbhg'cZYUWX'ch\Yff@'kcf\_"'H\]g'UddfcUWX']g'XYg][bYX'hc'XYjY'cd'hckUfXg'kcf\_!fYUX]bYgg'h\Y'VUg]WX][]hU''ci fbU']ga 'g\_]``g'ghi XYbhg'\Uj Y'UW&i]fYX'h\fci [\'dfYj]ci g'WcfY'Wci fgYg''=ik]``'dfcj]XY'mci 'k]h\'h\Y cddcfhi b]mhc'Udd`mh\YgY'g\_]``g']b'U'"fYU'kcfX"gYhJb[žk\YfY'h\Ymk]``'VY'a YbhcfYX'VmUb']bXi ghfm dfcZYqg]cbU'"

## •

### 7ci fgY @YUfb]b[ Ci hWca Yg

- %": cfa i `UhY`X][ ]hU``^ci fbU`]ga 'ghcfmi]XYUg`UbX`Yj U`i UhY`h\Y]f`"bYk gk cfh\]bYgg#di V`]g\UV]`]hm´ž`[ ]j Yb dUfh\W``Uf`hcd]\WUfYUg`UbX`]bhYbXYX`Ui X]Yb\Wq"
- &" D`UbžfYgYUfW\žWfYUhYžYX]h'UbX``Umci h'di V`]g\UV`Y`a i `h]!a YX]U`X][ ]h'U``'ĉi fbU`]ga "
- ' " K cf\_'Wc``UVcfUhjj Y`mžYh\]WJ``m'UbX`]b`UWWcfXUbWY`k]h\`a YX]U``Uk 'hc`WfYUhYžYX]hždi V`]g\`UbX di V`]W]gY`X][]hJ``ci fbU`]ga "
- ("Di V`]WJgY`X][]HU``^ci fbU`]ga `cb`gcWJU``a YX]U`d`UhZcfa gži gY`XUHU`UbU`mt]Wg`hc`HfUW\_`\]hg`cb di V`]g\YX`a UhYf]U`ža UbU[Y`fYUXYf`Wca a Ybhg`UbX`a UbU[Y`gYUfW\`Yb[]bY`cdh]a]gUh]cb`cZ U`di V`]WUh]cb`g]hY"

## HYUWX]b[ 'GhfUhY[]Yg'

H\Y`WcifgY`k]``VY`WcbXiWfYX`Ug`'!\cif`gYa]bUfg`Xif]b[`k\]WX`ghiXYbhg`k]``XYjY`cd`X][]hU```cifbU`]gadfc^YWfg`Zcf`dchYbhjU``diV`]WUhjcbžWc``UVcfUhjY`mgiV!YX]h#fYj]Yk`WUggaUhYgfWcbhYbhžXYg][b`kYVdU[YgZcf`h\Y`XY`]jYfmcZWcbhYbhžUbXž]b`WUgYg`k\YfY`kcf\_`\Ug`VYYb`diV`]g\YXžaUbU[Y`UbX`acb]hcf`]hg`idhU\_Ycb`gcWJU``aYX]U"

9UW. 'gYa ]bUf'k ]``'YggYbhJU``m'VY'U'X][ ]hU`'&i fbU`]ga '`'W]b]W"''HYUW.]b[ 'k ]``'df]a Uf]`m'hU\_Y'h\Y'Zcfa 'cZff\L'U WcbhJbi ci g'dfcW/gg'cZgi V!YX]hcf]U`!ghmY'ZYYXVUW\_#[ i ]XUbW/'Vmh\Y'hi hcf'fUb']bXi ghfmdfcZ/gg]cbU'\L']b'h\Y fc`Y'cZYX]hcf'UbX'f\L\L'\X'Wc``UVcfUhJj Y'fYj ]Yk ]b[ 'Vmghi XYbhg'cZYUW. 'ch\Yfftjik cf\_"''

H\]g'UddfcUW\']g'XYg][ bYX'\c'XYj Y`cd'\ck UfXg'k cf\_!fYUX]bYgg'\k\Y'VUg]WX][ ]\ti\'\ci fbU`]ga 'g\_]``g'gh XYb\g\\Uj Y'UW\ei ]fYX'\k\fci [\'dfYj ]ci g'WcfY'Wci fgYg"'\dik ]``'dfcj ]XY'\k\Ya 'k ]\k\'\k\Y'cddcfh b]\m\c'Udd`m\k\YgY g\_]``g']b'U'"fYU`\k cf`X"'gY\\b|[ žk\\YfY'\k\Ymk ]``'VY'a Yb\cfYX'VmUb']bXi g\fmdfcZ\Ygg]cbU`"

# 5ggYgga Ybh

# 5ggYgga Ybh'HUg\_g

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
Dcfhzc`]c'cZHi hcf]U` DfYdUfUh]cb'UbX'=b!hi hcf]U` DfcXi Wh]cb'HUg_g'	&\$I	Bch5dd`]WUV`Y	%ž'&ž' ž'(
A YX]U 7 cbhYbhDfYdUfUh]cbž			

# 5 https://bxubwww.Fyei]fYaYbhg

 $5\,hhYbXUbWf^{"}]g^{"}fYei~]fYX^{"}Uh^{"}U^{"}^{"}gYa~]bUfg^{"}fhhYfY^{"}UfY^{"}bc^{"}YWh~fYgL^{"}!~Uh^{"}U^{"}a~]b]a~i~a~^cZ,~\$^{"}dYfWfbh'^{"}bh^{"}hh^{"}yhyhh^{"}hh$ 

# 7ci fgY'GWXYXi `Y

J]Yk 'WUgg'ha YHUV'Y

H]a YHJV`Y

8UhY	Hmd Y	Equip by
K YY_'%'%)': YVfi n YVfi Ufm	Ufmi! '% GYa ]bUf	
		<b>=ÐffcXiVMjcb</b> "hc"h\Y"VciUfigY cj Y"c fc c
		FYj]Yk 'cZ'@W' ei adfZk'!'fUb[Y'cZUfh]WYgž'b'

K YY_''.'%'A UfWX'!') A UfWX	GYa ]bUf	O WW] k dWh[W aX [eegWe [ 'aUS^S`V [ fWd Sf[a`S^_ WW [S UahWdSYWaXbgT (U SXS [del eaU[S^fdW Ve S`V babg 'SdUg 'fgdWdQ  A UbU[ ]b[ 'U'Wda d`YI 'cb`]bY'di V`]g\]b[ 'd`UhZcfa '! k fUb[`]b[ 'h\Y'F YVY`A ci gY'VUW_YbX'Ug'U Wdbff]Vi hcf'UbX'gi V!YX]hcf  7 cdmYX]h]b[ ']ggi Yg'! '\ck 'UbX'k \Uhhc WdffYWfMXUb[ Y'k \Yb'gi V!YX]hb[ /a U_]b[ 'W'hg'UbX fYk f]h]b[ /]ggi Yg'cZghmY/Ybgi f]b[ 'fWYUbfi'Wdm'/k f]h]b[ '\YUX`]bYgz'ghUbX!Z]fghg'UbX'Wfcgg!\YUXg  GhcfmXYj Y`cda Ybhi dXUhY#Ydcfh  H\Y'@W' ei adfZk'dcXWUgh
K YY_'(.`, 'A UfWX'!'%& A UfWX	GYa ]bUf	O WW] k dWh[W aX [eegWe [ *aUS^S`V [ fWd Sf[a`S^_ W[S UahWdSYWaXbgT (U SXS[deł eaU[S^fdW Ve S`V babg *SdUg *fgdWdQ  GcW]U``a YX]U`Yb[ U[ Ya Ybh! dfca ch]b[ 'mci f'ck b UbX#cf'ch\Yf'Ui h\cffg`WcbhYbh  Di V`]g\]b[ 'WcbhYbhhc': UWYVcc_'UbX' ≠bghU[ fUa / GYUfW\'9b[]bY'C dh]a ]gUh]cb/Hk YYh]b[ 'mci f'WcbhYbh I dXUhY'cb'ghcfmXYj Y`cda Ybh  H\Y @W ei adfZk'dcXWUgh
K YY_').'%) 'A UfWX'!'%- A UfWX K YY_'*.'&&'A UfWX'!'&* A UfWX	GYa ]bUf	O WW] k dWh[W aX [eegWe [ 'aUS^S`V [ fWd Sf[a`S^_ W [S UahWdSYWaXbgT (U SXS [del eaU[S^fdW Ve S`V babg 'SdUg 'fgdWdQ  8][ ]hU`WcbhYbhdUW_U[ Y`f7cf`UggYgga Ybhz! Z]bU`]gUh]cb`UbX`gi Va ]gg]cb`W]b]W  GYUfWX`9b[ ]bY`Cdh]a ]gUh]cb`fVUg]Wg_]``gz  5#6`hYgh]b[ H\Y`@W ei adfZk`dcXWUgh : `YI`k YY_!`bc`gYa ]bUf
K YY_"+."&- "A UfWX"!"& 5df]`	GYa ]bUf	0 WW] 1k dWh[W aX [eegWe [ 'aUS^S`V [ fWd Sf[a`S^ _ WW[S UahWdSYWaXbgT1[U SXXS[del eaU[S^fdW Ve S`V babg1SdUg1fgdWdQ

		F Yj ]Yk 'cZ'X][ ]HU`WcbHYbhdUW_U[ YfgŁZfca 'YUf]Yfk YY_g  Gi VV]b[ 'UbX'`Um]b[ 'ci hmci f'WUgga UHYgfWcdm! hck UfXg'di V`]WUr]cb!gHUbXUfX'VYbW.a Uf_g  Dfca ch]b[ '@W' ei adfZk'WcbHYbh!']b!WUgg'Yl YfW]gY  9b[ U[ ]b[ 'U'fYUXYfg\]d'H\fci [ \'cb']bY'fbYk g'YHHYfgfi fh\Y @W' ei adfZk'bYk g'YHHYfŁ  8][ ]HU`'WcbHYbh'VfU]bghcfa ]b[ '!'mci f'bYl hX][ ]HU` dUW_U[ Y
K YY_', .') '5df]`'!'- '5df]`	GYa ]bUf	O WW] & dWh[W aX [eegWe [ 'aUS^S`V [ fWd Sf[a`S^_ W [S UahWdSYWaXbgT (U SXS [deleaU[S^fdW Ve S`V babg 'SdUg fgdWdQ  D]hW]b[ 'mci f'bYI hX][ ]hU`WcbhYbhdUW_U[ Y  8 ]gW gg]cb cZfXUhUfi'ci fbU`]ga žXUhU j ]gi U`]gUh]cb UbX ]bZc [ fUd\]Wg  H\Y @W ei adfZk fbYk g`YhhYffi  H\Y @W ei adfZk dcXWUgh
K YY_''%&'5df]`'!'%* 5df]`	GYa ]bUf	① WW] 1k dWh[W aX [eegWe [ 'aUS^S`V [ fWd Sf[a`S^_ W[S UahWdSYWaXbgT (U SXS [del eaU[S^fdW Ve S`V babg 'SdUg fgdWdQ  : ]bU`]gUh]cb`cZX][ ]hU`dUW_U[ Y`!`X]gW gg]cb`UbX ZYYXVUW_cb`XfUZrg#ZfghW hg`YhW  H\Y`@W ei adfZk dcXWUgh  H\Y`@W ei adfZk'bYk g`YhYf  F Ydcfhcb`dfYj ]ci g`gcWJU`!a YX]U`dfca ch]cb`cZ @W ei adfZk WcbhYbhfl UWYVcc_ž'+bghU[ fUa UbX`G9C Ł
K YY_'%\$.'% '5df]`'!'&' 5df]`	GYa ]bUf	WW] & dWh[Wi aX [eegWe [ 'aUS^S`V [ fWd Sf[a`S^ _ WW [S UahWdSYWaXbgT (U SXS [del eaU[S^fdW Ve S`V babg Sd Ug fgdWtQ  H65 .

## FYgci fW/g

DfYgWf]VYX F Ygci fWyg

Bc'hYlhVcc\_'Zcf'h\]g'WtifgY

## FYWca a YbXYX FYgci fWYg

Mci 'k ]``bYYX'hc'VY'fYUX]b[#]Yk]b[#]ghYb]b['UWfcgg'U'k]XY'j Uf]YhmcZ'ci fbU`]ga "

< YfY 'UfY 'gca Y 'gi [ [ Ygh]cbg"

@cb[Zcfa "cf[ža YX]i a "Wca ž'h\YWcbj YfgUh]cb"Wca ž'h\Y`\i Z]b[hcb'dcghž'H\Y'; i UfX]Ub'5i ghfU`]Už G`UhY"Wca ž'YhW"YhW

 $H\Y`GUhi fXUmDUdYfz"; ccX`K YY_YbX`UbX`BYk g`FYj ]Yk z`GdYWfi a `UbX`HfUj Y`Yf`gYWf]cbg`]b`h\Y`GmXbYm A cfb]b[ `<YfU`X`k YY_YbX`YX]h]cb`H\Y`K YY_YbX`5i ghfU`]Ub`a U[ Un]bY// <math>=$ bei ]fYf`UbX`FYj ]Yk `]b`H\Y K YY\_YbX`5i ghfU`]Ub`@]ZY`UbX`@Y]gi fYz

DYfgdYWfj Y`]b'h\Y`5i ghfu`]ub': ]bUbW]u``F Yj ]Yk'' **‰** J Wh '[ hc b'@]ZY'UbX'@Y]gi fYž

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g Hi fb]h]b Gi Va ]gg]cb

⊋mci 'YbWci bhYf'U'dfcV`Ya 'k \Yb'UhhYa dh]b[ 'hc'gi Va ]h'mci f'Ugg][ ba Ybh'h\fci [ \'Hi fb]h]bž'd`YUgY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

- i bXYfgHUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbhUhI BGK
   VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hz'I BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5VWWdhUV`Y'I gY'cZ IBGK =7 HF Ygci fWrg Dc`]Wh

# 5 WUXYa]W + DZcfa Uh]cb

: cf'YggYbhJU`ghi XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va ]gg]cbg'[i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/I BGK '<YU'h\ 'UbX'GUZYhmdc']W]Yg/</li>