

ARTS3065

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 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZIBGK']g`cWJtYX''$

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7 ci fgY @YUfb]b[Ci hWca Yg

%" 7 f]h]wU``mYb[U[Y`k]h\ XYVUhYg`UbX`]bHY``YWh U``dUfUX][a g`]b`Z`a `ghi X]Yg" &" @cWUhYz`cf[Ub]gYz`UbX`UggYgg`]bZcfa Uh]cb`fY`Yj Ubhihc`h\Y`ghi XmcZgWfYYb`a YX]U" \" 7 ca a i b]\WhY`h\Y`ci h\\can Yg`cZmci f`fYgYUf\X`]b`Vch\ `g\Xc`Uf`mUbX`bcb!gdY\JU`]gh``Ub[i U[Y" (" 8 Ya cbghfUhY`\Vf]h]\WD``g_]``g`]b`dfcV`Ya `gc`j]b[`UbX`gY`Z\X]fY\WhYX``YUfb]b["

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GWZC`XYX``YUfb]b[`]g`W\bfU``rc`t\Y`t\YUW\]b[`glfUlt\[]Yg`XYd`cm\X`]b`l\]g`W\cifgY"`5g`U`WJdghcbY`WcifgY 5F HG'\$*)`df]cf]hjgYg`Zcfa Utjj Y`ei Yghjcb]b[`zcb[c]b[`X]U`c[i Y`UbX`dYYf!hc!dYYf``YUfb]b[`UbX`t\YUW\]b[\zeta YUW\cZk\]W\d`\d`UmU`_YmdUfh]b`t\Y`k YY_`mgYa]bUfg"``7 ci fgY`a Utl\Yf]U`g`UbX`WUgg`X]gW`gg]cbg`UfY XYg][bYX`hc`W\U``Yb[Y`ghi XYbhg`t\fci [\`XYYdYf``YUfb]b[`UbX`X]gW`gg]cb`UbX`hc`YbUV`Y`ghi XYbhg`hc`a cj Y VYmcbX`t\Y]f`W`ffYbhg_]```UbX`_bck`YX[Y``Yj Y`g"'\d`t\]g`Wci fgY`UggYgga YbhtUg_g`UbX`hYUW\]b[`ghfUlt\[]Yg UfY`XYYd`m]bhYftk]bYXzk]h\`ghi XYbhg`\Uj]b[`t\Y`cddcfhi b]hmhc`XYj Y`cd`UbX`k cf_g\cd`t\Y]f`]XYUg`UbX dfc`YWg`t\fci [\`g\WZC`XYX`Zcfa Utlj Y`UbX`gi a a Utlj Y`UggYgga YbhtUg_g"

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F YgYUfVX 'DcfhZc`]c'	')।	&) #\$+#&\$&%`%%\$\$`DA	%ž'&
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K f]hhYb ZYYXVUW_cb k f]h]b['hUg_'UbX 'U'bi a Yf]\W\`[fUXY'j]U'\ci fgY @A G"

5XX]h]cbU`XYhJ]`g.

H\Y`Z]fgh'UggYgga Ybh'HUg_`]g`lk c`g\cfhk f]hhYb`d]YWYgž'Wca d`YhYX`]b`WUggž'VUgYX`cb`Wci fgY`a UhYf]U`g ffYUX]b[g'UbX`gWfYYb]b[gŁk YY_g`%' "'8YhU]`YX`]bZcfa Uh]cb`UVci h'h\Y`HUg_ž']bWi X]b[`gUa d`Y`ei Ygh]cbg UbX`a Uf_]b[`Wf]hYf]Užk]``VY`Uj U]`UV`Y`cb`A ccX`Y"

Gi Va]gg]cb bchYg. 7ca d`YhYX`]b!WUgg

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8 YW] g.

&\$\$\$ k cfX`YggUmcf`a]I YX`a YX]U`k cf_k]h\`UWWca dUbm]b[k f]hhYb`Wca a YbhUfmcZ) \$\$`k cfXg`h\Uh YI d`cfYg`U`ei Ygh]cbžWcbWYdhcf`XYVUhY`YI Ua]bYX`]b`h\Y`Wci fgY"

K f]hhyb ZyyXVUW_UbXbi a Yf]wb`[fUXYj]U'wci fgy @A G

H\]g`]g`h\Y`Z]bU``UggYgga YbhHJg_`Zcf`h\Y`Wci fgY"

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H\]g`]g`mci f`Z]bU``UggYgga Ybh'HUg_`Zcf'h\]g`Wci fgY''`Nci f`fYgYUfW\`dfcYWfik]``YI d`cfY`U'ei Ygh]cbz`WcbW'dh cf`XYVUhY`YI Ua]bYX`]b`h\Y`Wci fgY'h\fci [\`h\Y`WcgY`UbU`mg]g`cZcbY`cf'a cfY`Z]`a `cf'hY`Yj]g]cb'hYI hg fY`Yj Ubh'hc'h\Y`Wci fgY'hcd]W'`±bZcfa Uh]cb`UVci hh\Y`fYgYUfW\`dfcYWf]bWi X]b[`[i]XUbW'`cb'hcd]Wg`UbX UggYgga Ybh'Wf]hYf]U'k]```VY`Uj U]`UV`Y`cb'A ccX`Y"'H\Y``Ugh'k YY_`cZh\Y`Wci fgY`]bWi XYg`U dfcYWfik cf_g\cd']b`k\]W\`k\Y`k\]``k\cf_g\cd'k\cf_j]b`dfc[fYgg"

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 $\label{thm:compact} $$H\YfY^{g}bc^{h}lh^{g}w^{i}fgY^{!!}h^{Y}YggYbh^{l}U^{T}YUX]b[g^{z}f^{Y}UW^{k}YY_{k}]^{u}VY^{U}j^{U}]^{u}VY^{c}b^{A}ccXY^{h}fci[\wwfgy^{g}Y^{g}Y^{h}QY^{g}Y^{h}QY^{g}Y^{h}QU^{T}Zcf^{h}Y^{g}W^{g}Z_{k}]^{u}V^{g}Z_{k}]^{u}VY^{g}Y^{g}Y^{h}QU^{T}Zcf^{h}Y^{g}W^{g}Z_{k}]^{u}VY^{g}Z_{k}]^{u}VY^{g}Y^{g}Y^{h}QU^{T}Zcf^{h}Y^{g}W^{g}Z_{k}]^{u}VY^{g}Z_{k}]^{u}Z_{k}]^{u}VY^{g}Z_{k}]^{u}VY^{g}Z_{k}]^{u}VY^{g}Z_{k}]^{u}Z_{k}]^{u}Z_{k}]^{u}Z_{k}$

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· i

D`YUgY`bchY`h\]g`mYUf`kY`k]```U`gc`fi b`cdh]cbU``gWfYYb]b[`XUmg`Zcf`]bhYbg]j Y`Zj`a `j]Yk]b[`UbX`X]gW gg]cb'!!

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
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a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
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; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

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D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

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