

## **ARTS3047**

7 cbhYa dcfUfm7 f]h]WU``UbX`7 i `hi fU``H\Ycfm

HYfa 'H\fYY''##'&\$%

### 7ci fgY'Cj Yfj ]Yk

#### Ghuzz 7 cbhuWh8 Yhu]`g

#### '7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
G][] <sup>-</sup> >chh_UbXh	g"1ch_UbXh4 i bgk "YXi "Ui	H\i fgXUmgʻUh'‰	K YVghYf" %%>	l)*,%

#### GWXcc~7cbhJWh~abZcfa Uhlcb

Fcca " &ž`Yj Y`" FcVYfhK YVghYf'6i ]`X]b[

D\cbY.'f\$&Ł'-',)'(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]tjcbU``7igtcX]Ubg\cong h\Y'6YXY[U'`fPYbg]b[tcb'WJadig\cong \cong m1]U'`fP]thubX'5fti/'8Yg][b'7UadigYg\cong \cong tUbX't\Y'B[ibbUkU`dYcd'Y'fbigffU']Ub'8YZYbW': cf\W'5\WJXYam]b'7UbVYffU\cozh\Y'`UbXg'k\YfY'YUW\'WJadig'cZl\BGK']g`c\WJHX''$ 

#### 7ci fgY 8YW] g

7 fYX]hDc]bhg \* \*

#### Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. 7`Y/eZ

 $H\[g'WcifgY']bffcXi\ W'g'mci' hc'gca\ Y'cZh\Y'WbffU'hYI hg'UbX'WcbW'dhg']b'h\Y'_Ym']bhY``YWh U``a cj Ya\ Ybhg UbX'h\Ycf]Yg'Zfca 'h\Y'YUf'mfk Ybh]Yh\ 'W'bhi fmhc'h\Y'dfYgYbh'Ug'h\YgY'VYUf'cb'']hYfUfm'UbU`mg]g'''Mci'k ]``YI Ua ]bY'U'fUb[Y'cZfY`UhYX']qqi Yg']bWi X]b[.$ 

%"\ck 'h\Y'j YfmUggi a dh]cbg'cZ`Ub[i U[YžWca a i b]WUh]cb'UbX'a YUb]b['\Uj Y'VYYb'ei Ygh]cbYX'Zfca 'YUf'm k Ybh]Yh\'Wbh fmhc'h\Y'dfYgYbh'

&"'h\Y'fY!WcbWdh]cb'cZ]XYbh]mmUbX']rg']a dUWhcb'`]hYfUfmWcbglfi Wf]cb/

'"'h\Y`k Umg`dc`]h]WU`ghfi Whifyg`Wcbghfi Whih\Y`fY`Uh]cbg\]d`VYhk YYb`\i a Ub`gi V^YWhg`UbX`cV^YWhg`gi W\`Ug Wca a cX]h]Yg"

#### 7 ci fgY @YUfb]b[ Ci hWca Yg

%" XYd`cmih\YcfYh]WU``hYl hg`]b``]hYfUfmiUbU`mg]g &" Wca dUfY'j Uf]ci g'h\YcfYh]WU``dcg]h]cbg`UbX`UddfcUVXYg ' " 'WcbXi WhifYgYUfVX`]b'h\Y`UfYU'cZWcbhYa dcfUfmWf]h]WU``h\Ycfm ( " k f]hY`U'WYUf`UbX`Zi YbhYggUmYb[ U[ YX`k ]h\ 'h\YcfYh]WU``]ggi Yg ) " k cf\_`]bXYdYbXYbhmUbX`]b'Wc``UVcfUh]cb

#### HYUWX]b[ 'GhfUhY[]Yg'

H\]g`Wci fgY`]g`]bZcfa YX`VmUb`UddfcUW\ hc`hYUW\]b[`UbX``YUfb]b[`h\UhZcghYfg`]bhYfUWfj`]mz`gh XYbhU[YbWn UbX'h\Y'd`YUgi fY'cZ\_bck 'YX[Y"'=hdfca chYg'bYk 'k Umg'Zcf'gh XYbhg'hc XYj Y`cd'dfcXi Whj Y'dcgg]V]]hJYg Zfca 'th\Y'X]qWld`]bUfmVUgYg'cZth\Y]f'gh XmUbX'tc'fYWc[b]qY'th\Y'dctYbhlu`]b']bHYfX]qWld`]bUfmYb[U[Ya Ybh]b h\Y`\ia Ub]h\Yg":5q'U'h\]fX'm\YUf'Wci fqYž']hdfca ch\Yg']bXYd\YbX\Ybh'\YUfb]b[ 'Ug'f\Z\YW\X']b'gh X\Ybh\ZcW'gg\X gYa ]bUfgžh\Y'Ya d\Ug]g'cb'fYgYUfW\žUbX'h\Y'Wci fgY'gma dcg]i a "'H\YfY'k ]``VY'U'cbY!\ci f``YWfi fY'YUW\ k YY\_".5 "YWn fY ]g'a cfY h\ub'U'dfYgYbhUhcb'cZ]bZcfa Uhlcb/]h]g'Ub'UWnj Y'UbX ]bhYfUWnj Y'Zcfi a 'k \YfY ]XYUg`UbX`fYUX]b[ 'dfUWf\W'g`UfY`fY\YUfgYXž'hYqhYX`UbX`VXU``Yb[ YX"'=h`]g'h\Y`g]hY`k\YfY`gh XYbhg YbWci bhYf'h\Y`Wcbj YbhJcbg`cZdfYgYbhJb[`UWUXYa]WfYgYUfW\`UbX`ghU[]b[`UWUXYa]WXYVUhY''@YWhffY UthYbXUbWY]grgi ddcftYX:Vmtk c fYj ]g]cb tcc`g. YWkc' \*\$"UbX"cb!`]bY'a UtYf]U``dcgtYX`]b th\Y'k YY\_"UZtYf'th\Y `YWn fY"`BY]h\Yf`cZh\YgY`gi ddcfhg`]g`U`fYd`UWYa YbhZcf`UhYbXUbWY"`H\Y`YVXc' \*\$`]g`U`fYWcfX`cZUb`Yj Ybh bchu'gi a a UfmcZ]bZcfa Uficb" + YYd]b[ k ]h hYcVYWfi YgcZh]fX!mYUf gh XmzhYgYa ]bUfa cXY dfcj |XYg'U'a cfY'|bXYdYbXYbhUbX'Wc``UVcfUhj Y ``YUfb]b[ 'Ybj |fcba Ybh''K \]Y'Z ``mgi ddcfhYX'UbX'X]fYWhYXž h\Y`gYa]bUf`U``ckg`ghiXYbhg`hc`UggiaY`]bWfYUgYX`fYgdcbg]V]`]hmZcf`h\Y`X]fYWf]cb`cZYUWX`WUgg`UbX`Zcf`h\Y Zcfa i 'Uhlcb'cZfYgYUfW.'Hug g"'H\YmUfY'U'gc'UV'Y'hc'WflhWu'`mfYZYWficb'h\Y]f'ck b'dfcWyggYg'cZ'YUfb]b[ž k\]WX']g'cZdUff\W'\Uf']a dcfrub\W']b'U'\Ci fqY']bfcXi Wb['bYk'\Wf\\W\TfYUX]b['dfU\W\\W'g"'H\Y'gYa]bUf fYei ]fYg YUWX ghi XYbhhc i bXYfhU\_Y. dfYdUfUh]cbž ]bWi X]b[ gca Y Wc ``UVcfUh]cb k ]h\ ch\Yf ghi XYbhg/ X]gW gg]cb`UbX`dfYgYbfUh]cbg/Wc``UVcfUh]j Y`k cf\_`]b`ga U```[ fci dg"`Hck UfXg`h\Y`YbX`cZh\Y`Wci fgY`k Y`k ]`` \c`X`U`h\Ya YX`Gma dcg|i a `|b`d`UWY`cZgYa |bUfg"`DfY!UffUb[YX`[fci dg`k]```YUW\`[]i Y`U`&`a |bi hY dfYgYblUhcbicbiUlhcd]Wh\Uhl\UgiVYYbiUddfcjYXiVmh\YiWbjYbcf"H\]gik]``YbUVYighiXYbhgihciYb[U[Y k]h\rh\Y`Z\congruential fightham for the fightham for the

#### 5ggYgga Ybh

A midfYZYffYX'fYZYfYbWJb['ghmY'Zcf'h\Y'A U'cf'YggUm']g'A cXYfb'@Ub[i U[Yg'5ggcWJUh]cb'fA @5'ghmYŁ": cf Yl Ua d'Yg'cZ\ck 'hc'i gY'h\]gžgYY

\hdg.#ghm\Y"a \U"cf[ #k cf g! W|hYX!U!ei ]W![i]XY#

\hd.#|\VfUfm'\Vcb\VcfX|\U"\VU#\Y`d#\\h|b[#a `U"d\d

.

#### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'tUg_	K Y][ \h	8iY'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
6`c['FYZYW¶cb'HJg_'	&\$I	\$, #%#&\$% `\$).\$\$`DA	%ž&Ž Ž)
Gma dcg]i a 'DfYgYblUljcb'	' \$ı	%( #%%\$\$% `\$+.\$\$`DA	%ž&ž ž)
A UʻcfʻYggUmi	) \$1	&-#%#&\$%-`\$).\$\$`DA	%ž&ž ž( ž)

#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh'% '6`c[ 'FYZYWh]cb 'hUg\_

GhUfh'XUhY.

@Yb[h\.'%\$\$\$

8 YHJ]`g. '%\$\$\*k cfXg`]b`hcHJ`"@cWlY`U`W`hi fU``cf``]hYfUfmiYl h'UbX`k f]hY`U`fYZYWjcb`cb`]hžYl d`cf]b[ `\ck `]h
]`i ghfUhYg`cbY`cZh\Y`h\YcfYhJWJ``WcbWYdhg`Zfca `h\Y`Wci fgY"`8 c`h\]g`Zcf`h\fYY`X]ZZYfYbhk YY\_`mhcd]\Wg"`\Mci
a UmbchV`c[ `cb`h\Y`hcd]\WcZmci f`Gma dcg]i a `dfYgYbhUh]cb`Vi ha UmV`c[ `cb`h\Y`h\YcfYhJWJ``WcbWYdhmci
XYU``k ]h\`]b`mci f`a U'cf`YggUm'Gi Va ]gg]cb`bchYg. `GhU[ [ YfYX`gi Va ]gg]cbg`cb`Wci fgY`fYUXYf`V`c[ `Zfca
k YY\_`&"K f]hYb`ZYYXVUW\_UbX`bi a Yf]\WJ``[ fUXY`j ]U`@A G"

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh& Gma dcg]i a DfYgYbhJh]cb

GhUfh'XUhY. 1%+#%\$#&\$%- 1\$-.\$\$ 15A

8YHJ]`g. `H\Y`Gma dcg]i a `]g`U`a ]b]!\%cbZYfYb\W`fi b`Vmh\Y`ghi XYbhg`fck UfXg`h\Y`YbX`cZh\Y`\\chi fgY"
Ghi XYbhg`UfY`cf[ Ub]gYX`]bhc`[ fci dg`UbX`a YYhcb`]bY`UbX`ZU\W`hc`ZU\W`hc`dfYdUfY`h\Y]f`dfYgYbhUh]cb`cb`U
hYI h\Z̄ca `h\Y`Z]bU``a cXi `Y`]b`h\Y`\\chi fgY"`H\]g`]g`h\Y`Z]bU``UggYgga YbhHUg\_"`Ghi XYbhg`fY\\]j Y`U`a Uf\_`g\YYh
h\UhXYhU]`g`hk c`[ fUXYg.`cbY`Zcf`h\Y`[ fci d`k cf\_`Ug`U`k \c`Y`UbX`cbY`Zcf`h\Y]f`]bX]j ]Xi U`
\\chi bhf]Vi h]cb"K f]hhYb`ZYYX\U\W\_`UbX`bi a Yf]\\U\`[ fUXY`j ]U`@A G"

#### 5XX]h]cbU`XYhU]`g.

 $H\setminus g'mYUfžkY'k]^*'\setminus c^X'kc^7f]h]WU^H\setminus Ycfm'Gmadcg]U''$ 

D`YUgY`bchYžmci `UfY`cb`mfYei]fYX`hc`dfYgYbhUhcbY`cZh\Ya žY]h\Yf`Uhih\Y`gYa]bUf`cZ%+h\`C WhcVYfž`cf`Uhh\Y`gYa]bUf`cZ%(h\`Bcj Ya VYf"

5ggYgga Ybh' . 'A U'cf'YggUm

GhUfh'XUhY.

**8YHJ]`g**. `&) \$\$`k cfXg"`H\]g`]g`h\Y`Z]bU``UggYgga YbhiHJg\_"Ghi XYbhg`k]```fYWY]j Y`k f]hhYb`ZYYXVUW\_`UbX`U bi a Yf]WU``[ fUXY"

 $\label{thm:condition} \mbox{Hi fb]h]b"gYh]b" Label Carlo Carlo$ 

# F Ygci fWYg DfYgWf]VYX F Ygci fWYg

A cghcZh\Y'fYei ]fYX'hYl hg'UfY'Zfca 'h\Y'

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

\_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k ]``

#### 5 WUXYa | W < cbYghmUbX D`U[ | Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU\_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW\_bck iYX[Ya Ybh'i

IBGK '[fcidg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

## 5 WUXYa ]W±bZcfa Uhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/