

# **ARTS3124**

7 c``UVcfUhjj Y'DYfZcfa UbWY'A U\_]b[

HYfa "H\fYY" ## "&\$%-

# 7ci fgY 8YHJ] g

7 fYX]hDc]bhg \*

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. FZWSfdWS` V BWdAd\_S`UWEfgV[We

7 c``UVcfUhj Y'DYfZcfa UbW'A U\_]b[']bj]hYg'mci 'hc'k cf\_'hc[Yh\Yf'k]h\ 'ch\Yfg'UWcgg'U'fUb[Y'cZX]ZYfYbh WfYUhj Y'Zcfa g'hc'XYj Y'cd'Ub'cf][]bU'`&\$!a]bi hY'ga U``![fci d'dYfZcfa UbW'žk\]W\ 'k]```\Uj Y'U'di V`]W dfYgYbhUhjcb'Uhih\Y'YbX'cZhYfa 'gi ddcfhYX'Vmh\Y'7 fYUhj Y'DfUWJW'@UV'''Mci 'k]```YY'YX'h\fci [\'kYY\_m k cf\_g\cdg'h\UhiW\U`Yb[Y'h\Y'ZJa]']Uf'UbX'Yl dUbX'mci f']a U[]bUhjcb'''Mci 'k]```YUfb'bch'\ghih\fci [\ hU`\_]b['UVci ha U\_]b[žVi hh\fci [\ 'UWhj Y`ma U\_]b['/UbX']b'Xc]b['gc'mci 'k]```gi fdf]gY'mci fgY'Zk]h\ k\Uh mci 'UfY'WddUYY'cZXfYUa]b['i d'''A cgh]a dcfhUbhmžmci 'k]``k cf\_'Wc``UVcfUhj Y`mk]h\ ch\Yfg'Ug'U'ghfYb[h\ UbX'fYgci fW'žVf]b[]b['mci f'X]j YfgY'ghcf]Ygža Ya cf]Ygžei Yghjcbgž'UbX'dUgg]cbg']bhc'fY`Uhjcb'k]h\ YUW\ ch\Yfz'UbX``YUfb]b['grfi Wf fYg'UbX'Zcfa g'h\Uhj U'i Y'ci f'X]ZYfYbW'g'Ug'k Y'WfYUhY'U'g\UfYX'h\YUhf]\W'`U'g\UfYX'h\YUhf]\W'``Ub[i U[Y'''H\]g'Wci fgY'XcYg'bchUggi a Y'UbmdUfh]W'`Uf'\_]bX'cZdYfZcfa UbW'Yl dYf]YbW'z'UbX']g'gi ]hUV'Y Zcf'U``k\c'UfY']bhYfYghYx']b'Yl dUbX]b['h\Y]f'WfYUhj ]mubX'XYj Y`cd]b['Wc``UVcfUhj Y'k cf\_]b['g\_\_'``g'h\Uh\Wb VY'Uda']Yx'hc'U'fUb[Y'cZWcbhYl hg''

### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" 5dd`mUddfcdf]UhY`h\YcfYh]WU``UbX`d\mg]WU``fYgYUfW\`dfUWf]WYg`]b`h\Y`WfYUh]cb`cZdYfZcfa UbW\ &" 9j U`i UhY`UbX`Wcbghfi Whjj Y`mWf]hjei Y`a UhYf]U``[ YbYfUhYX`]bX]j ]Xi U``mUbX`]b`[ fci dg`Ug`dUfhcZh\Y WfYUhji Y`dfcWYgq
- ' "Kcf\_`YZZYWNj Y`m`]b`gaU```[fcidgžXYjY`cd]b[`Wc``UVcfUNjY'g\_]``g`UbX`gY`ZfY`]UbWY`]b`d`Ubb]b[`UbX YIYWNjcb

## HYUW ]b[ 'GhfUhY[ ]Yg'

H\]g`Wci fgY`Wca V]bYg`Yl dcgi fY`rc`X]gh]bWh]i Y`Ufr]gh]WdfUWh]Wfg`UbX`WcbWfdrg`k]h\`Yl dYf]Ybr]U```YUfb]b[h\fci [\`gc`c`UbX`Wc``UVcfUh]j Y`a U\_]b["'=b`r\Y`Z]fghdUfhcZh\Y`a cXi `Yž'h\Y`k YY\_`mgYgg]cbg`U`hYfbUhY VYhk YYb`dfYgYbhUh]cb`cZa UhYf]U``UbX`[i]XYXYl YfW]gYg`Zfca 'h\Y`Wci fgY`Wcbj YbYfž'UbX`]bX]j ]Xi U``gc`c dYfZcfa UbWfg`mci 'a U\_Y`]b`cfXYf`hc`Yb[U[Y`k]h\`r\Y`Zcfa g`UbX`]XYUg`k\]W\`mci 'X]gWcj Yf"'=b`r\Y`gYWcbX dUfhcZh\Y`a cXi `Yžmci 'k]``k cf\_`]b`ga U```[fci dg`rc`XYj]gY`Ub`cf][]bU``dYfZcfa UbWfžk]h\`fY[i`Uf XfUa Uhi f[]WU`ždfUWh]WU`ž'UbX`WcbWfdh`U``[i]XUbWf`Zfca 'h\Y`Wci fgY`Wcbj YbYf"'

6i ]`X]b[ 'cb']bX]j ]Xi U``Wcbhf]Vi h]cbg'UbX`]bhYfYghgžmci 'UfY'gi ddcfhYX`]b`Z]bX]b[ 'WfYUh]j Y'a cXYg'cZ Wc``UVcfUh]cb—'h\Uh]gžmci 'k]``bchcb`mWc``UVcfUhY'hc a U\_Y'mci f'UfhžVi hk Y'a ][\hU\gc'h\]b\_'UVci hh\Y UfhcZWc``UVcfUh]cb']hgY`Z'`+b'h\]g'WUgYžmci f'Wc``UVcfUh]cb'k]```W`a ]bUhY`]b'U'di V`]WdYfZcfa UbWY'VYZcfY'U `]j Y'Ui X]YbWYžVi hh\Y'h\]b\_]b[ 'UbX'YI dYf]YbWY'k ]h\Wc``UVcfUh]cb'Wb'VY'Udd`]YX`]b'a Ubmch\Yf'WcbhYI hg"

# 5ggYgga Ybh

H\]g`WciffgY]gYWcii]fgYg`fnceii]fcYgHncbiXhc`UHh/b

- U'HYWXb]WU``FY\YUfgU``hc'\
  U'8fYgg'FY\YUfgU`ž')!-da Ž
  A mYfg

 $\label{thm:conditional} \begin{tabular}{ll} Hi fb]h]b"gYh]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \\ \begin{tabular}{ll} Hi fb]h]b"gYh]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"uf]hm fYdcfhg" \\ \begin{tabular}{ll} Hi fb]h]b"gYh]b"[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"uf]hm fYdcfhg" \\ \begin{tabular}{ll} Hi fb]h]b"ghi Ybhg"ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"uf]hm fYdcfhg" \\ \begin{tabular}{ll} Hi fb]h]b"ghi Ybhg"ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"uf]hm fYdcfhg" \\ \begin{tabular}{ll} Hi fb]h[h]b"g]a]hhYX"h\fci [\Thi fb]h]b"ugg]a]hhYX"h\fci [\Thi fb]h]hyX"h\fci [\Thi fb]hyX"h\fci [\Thi fb]hyX$ 

5ggYgga Ybh' .; fci d'dYfZcfa UbW

GhUfh'XUhY.

 $8YHU]^*g.^{+}:]bU^{+}[fciddYfZcfaUbWY^*Qk)+^{-}[fciddaUf_^*\check{Z}^*k)+^{-}]bX]_{j}]XiU^{+}Wcblf]Vih]cb''Q$ 

5 X X ] h] c b U` ` X Y h U ] ` g.

.

# FYgci fW/g

## DfYgWf]VYX F Ygci fWYg

 $\label{thm:continuity:continuity:de} $$HYfY'k ]^*VY'g\cfhVfYU_g'Uhgi ]^HUVY'dc]bhg'Xi f]b[ YUWX'k YY_`mWUgg"' gY'cZd\cbYg'UbX``Udhcdg'k ]^* bchVY'dYfa ]^HYX'Xi f]b[ WUgg"' 6f]b[ hc'YUWX'WUgg. YchY'cZk UhYf'UbX'U'gbUW_ž'Wca ZcfhUV'Y'Wch\Yg'Zcfa cj Ya Ybhž'UbX'U'bchYVcc_'UbX'dYb"$ 

## FYWca a YbXYX FYgci fWYg

D`YUgY`gYY`@Y[Ubhc`Zcf'h\Y``]ghcZfYWca a YbXYX`hYI hg`fY`UhYX`hc`YUWX`k YY\_fig`h\Ya Yž'k \]W\`k]```VY`i gYZ\` Zcf'h\Y`WUgg`Wf]h]ei Yg"

## 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

 $H\setminus g$  Wci fgY k ]``VY Yj U`i UhYX h\fci [\'U`A m9I dYf]YbWY gi fj YmUhh\Y YbX cZh\Y Wci fgY"

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

\_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k ]``

#### 5 WUXYa | W < cbYghmUbX D`U[ | Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU\_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci hWfYX]hUbX'hc d]YW]b[ 'hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb. 'k cf\_]b[ 'k ]th 'cth Yfg'Vi hidUgg]b[ 'cZZth Y 'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b[ 'mci f'k cf\_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

#### 5 WUXYa ]W = b Zcfa Uh cb.

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb[ 'hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua ]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybh fY/
- gh XYbhYei ]lmUbX X]gUV]`]lm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W']bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

#### a U[Y'7fYX]h

=a U[Y'Zfca '&\$% 'di V']Wg\Uf]b[ž;`Xf [fWH[W e"'D\chc'Vm<Y]Xfi b'@Ò\f"

#### 7F = 7CG

7F = 7CG Dfcj ]XYf 7cXY. \$\$\$-,;