

# 7ci fgY'Cj Yfj ]Yk

#### Ghuzz 7 cbhuWh8 Yhu]`g

#### '7cbj Ybcfg'

BUa Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
>]b[ 'N\Uc	3 .	<b>%</b> .\$\$!% .\$\$	Fa&' %8ž FcVYfh KYVgHYf	-',),\$**

#### GWkcc~7cbhJWh~abZcfa Uh]cb

Fcca " &ž`Yj Y`" FcVYfhK YVghYf`6i ]`X]b[

D\cbY.'f\$&E'-',)'(,)\*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.#gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]tjcbU``7i gtcX]Ubg\cong h\Y'6YXY[U'`fPYbg]b[tcb'WJa di g\cong tz'; UX][U'`fP]tmUbX'5fth/ '8Yg][b'7Ua di gYg\cong tubX't\Y'B[i bbUk U` dYcd'Y'fbi gtfU']Ub'8YZYbW': cf\w'5\\WXYa m']b'7UbVYffU\cozh\Y'`UbXg'k\YfY'YUW\cozh\Y'YUW\cozh\Y''UbXg'k\YfY'YUW\cozh\Y'' \cozh\Y''\UbXg'k\YfY'YUW\cozh\Y'' \cozh\Y''\UbXg'k\YfY'YUW\cozh\Y'' \cozh\Y''' \cozh\Y'''' \cozh\Y''' \cozh\Y'''' \cozh\Y''' \cozh\Y'' \cozh\Y'' \cozh\Y''' \cozh\Y'' \cozh\Y'' \cozh\Y''' \cozh\Y'' \cozh\Y'' \cozh\Y''' \cozh\Y'' \cozh\Y''' \cozh\Y'' \cozh\Y''' \cozh\Y''' \cozh\Y''' \cozh\Y''' \cozh\Y''' \cozh\Y''' \cozh\Y''' \cozh\Y''' \cozh\Y'' \cozh$ 

# 7ci fgY 8YHJ] g

## 7 fYX]hDc]bhg\*\*

### Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`k]``dfcj]XY`mci `k]h\`U`WcbhYa dcfUfm'UbX`Wf]h]WU``i bXYfghUbX]b[`cZh\Y`Z bXUa YbhU``WcbW'dhg UbX`dfcWfggYg`h\Uhi bXYfd]b`di V`]WfY`Uh]cbg`ghfUhY[]Yg`UbX`WJa dU][ bg"`H\Y`Wci fgY`ZcWi gYg`cb`ghfUhY[]W Wca a i b]WUh]cb`d`Ubb]b[ ždi V`]WfY`Uh]cbg`fYgYUfW\žh\Y`fc`Y`cZWi`hi fY`]b`di V`]WfY`Uh]cbgž]ggi Yg`UbX`Wf]g]g Wca a i b]WUh]cbž`WcfdcfUhY`gc\Y]U``fYgdcbg]V]`]hm'UbX`Yh\]Wg"

### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" → XYbhZmu fub[Y cZuddfcuWkYg ubx XYVuhYg uvci hhkY buh fYžfc Y ubx j u i Y cZdi V j WfY uhcbq
- ' " 5dd`miWca a i b]WUh]cb`UbX`glfUhY[ miWcbWldhg`hc`WUa dU][ b`d`Ubb]b[ 'UbX`dfYdUfY`di V`]WfY`Uh]cbg Wca a i b]WUh]cb`hYl hg`UbX`a UhYf]U`q
- ("5dd`miU`fUb[Y`cZYggYbh]U``hU\_YUk Umg`Zfca `Yh\]WU``di V`]WfY`Uh]cbg`dfUWf)WY`UbX`WcfdcfUhY`gcW]U`fYqdcbg]V]`]hm

### HYUWX]b[ GhfUhY[]Yg

H\]g'Wci fgY'k ]``Ya d`cmgYa ]bUfg'hc'dfcj ]XY'fY[i `Uf'cddcfhi b]hJYg'Zcf'dfcXi Wfj Y'X]U`c[i Yg'UbX Wc``UVcfUhJcb"'Ghi XYbhg'k ]``Yb[U[Y']b'ghfUhY[mXYj Y`cda Ybh'UbX'UbU`mg]gž'X]gW gg'WcbhYa dcfUfm gWbUf]cgž'UbU`mgY 'WugY'ghi X]Yg'UbX'XYj Y`cd'Wca a i b]\WhJcb'hYl hg"'H\Y'`YUfb]b['dfc\Wgg'k]``Ya d\Ug]gY 'fYU`'k cf'X''di V`]\WfY`UhJcbg'dfcV'Ya gž'W.U``Yb[Yg'UbX'cddcfhi b]hJYg'hc'Ugg]ghighi XYbhg'hc fY`UhY'h\YcfYhJ\U'\Wb\Wdfy'hc'h\Y'dfU\WfJ\W'cZdi V`]\WfY`UhJcbg'':\DhYfU\Wfj Y'UbX'fYZY\Wfj Y'hYU\X]b['UddfcU\XYg'k]``'VY df]cf]hJgYX'U`cb['k]h\'Xb\Wdhi U``ZfUa Yk cf\_g'UbX'Udd`]YX'a Yh\cXg''

# 5ggYgga Ybh

D`YUgY`gYY`a cfY`gdYWJZJW[i]XY`]bYg`UbX`gi ddcfhfYgci fWYg`cb`A ccX`Y"

## 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
FYdcfh	' \$ı	&\$#%\$#&\$% `%%) - `DA	%ž&Ž Ž(
Hi hcf]U`ZUWj]hUhjcb	&\$ı	dYf <sup>-</sup> gYa]bUf <sup>-</sup> gWkYXi <sup>-</sup> Y	%ž&
7 Ua dU][ b·7 f]h]ei Y·	) \$1	&(#%#&\$% <sup>·</sup> %%) - <sup>·</sup> DA	%ž&Ž Ž(

#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% F Ydcfh

GHJfh:XUhY.

**@Yb[h**\. %\$\$\$ k cfXg

**8YHJ]**`g. '%\$\$\$`k cfX`]bX]j ]Xi U``fYdcfH'': YYXVUW\_'j ]U'Hi fb]hJb"

5XX]h]cbU`XYhJ]`g.

D`YUgY`gYY`a cfY`gdYWJZJW[i]XY`]bYg`UbX`gi ddcfhfYgci fWYg`cb`A ccX`Y"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYh]b" . "H\]g"Ugg][ba Ybh]g"gi Va ]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a ]"Uf]hmfYdcfhg" \\ \end{tabular}$ 

5ggYgga Ybh& Hi hcf]U`ZUW]]HUh]cb

GhUfh XUhY.

@Yb[h\. '%s a ]bg dYf dYfgcb ]b U [fci d

8YHJ] g.; fci d'Ugg][ ba Ybh' % a ]bg dYf dYfgcb": YYXVUW j ]U'Hi fb]h]b"

5XX]h]cbU`XYhU]`g.

D`YUgY`gYY`a cfY`gdYWJZJW[i]XY`]bYg`UbX`gi ddcfhfYgci fWYg`cb`A ccX`Y"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$ 

5ggYgga Ybh' . 7Ua dU][ b 7f]h]ei Y

Ghufh'XUhY.

@Yb[h\."%) \$\$"k cfXg

**8YHJ]**`g. '%) \$\$ 'k cfXg": YYXVUW\_'j ]U'Hi fb]h]b"

5XX]h]cbU`XYhU]`g.

D`YUgY`gYY`a cfY`gdYWJZJW[i]XY`]bYg`UbX`gi ddcffifYgci fWYg`cb`A ccX`Y"

 $\label{thm:condition} \mbox{Hi fb]h]b"gYh]b" Label Label$ 

# FYgci fWfg

DfYgWf]VYX F Ygci fWYg

5j U]`UV`Y`cb`A ccX`Y"

FYWca a YbXYX FYgci fWYg

5j U]`UV`Y`cb'A ccX`Y"

## 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\]g`Wci fgY`k]```i bXYf[c'Wcbh]bi U``XYj Y`cda Ybhzij ]U'ZYYXVUW\_'dfcWYggYg'gi W\`Ug`ghi XYbhigi fj Ymg`UbX Wc``Y[]U``fYj]Yk"'K Y`h\YfYZcfY`hU\_Y`mci f`ZYYXVUW\_j YfmgYf]ci g`m''K Y`k]```U`gc'Wc``YWh]bZcfa U``ZYYXVUW\_]b`h\Y`Zcfa `cZUbcbma ci g`gi [[Ygh]cbg'UbX`Wca a Ybhg`h\fci [\ci hih\Y`Wci fgY"

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b[ 'rc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-', ) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''#c'ici
UfY'i bUV'Y 'rc'gi Va ]hmci f'Ugg][ ba YbhXi Y'rc'U'ZJi 'hk ]h\'Hi fb]r]b'mci 'a Um'Udd'mZcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'r]WYhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfgt'rc
]bWi XY'Ug'Yj ]XYbWY 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9I hYfbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ''mfYWY]j Y'U'r]W\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]'''bYYX'rc'gdYW]ZJWJ''mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mj ]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\ U`hYfbUhj Y'gi Va ]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va ]hUgq][ ba Ybha ccXiY

#### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'=hWJb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhU`mWcdm]b[ 'Zfca `U`gci fWY'k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]uf]ga ]bhch\YZc``ck]b[ WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[ .'7\Ub[ ]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][ ]bU`
ghfi Whi fY 'UbX#cf'dfc[ fYgg]cb'cZ]XYUg'cZh\Y'cf][ ]bU`ž'UbX']bZcfa Uh]cb'k ]h\ci h\UW\_bck `YX[ Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h\UhYX]hUbX'hc
d]YW]b[ 'hc[ Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k ]h\ci h\Uddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb.'k cf\_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b['mci f'k cf\_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa ]WhUg\_ž'ghYU']b['cf'UWei]f]b[ Ubch\Yf'dYfgcb'g'UWUXYa ]Wk cf\_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf\_'cf'gYY\_]b[ dUma YbhZcf'Wca d'Yh]b['UWUXYa ]Wk cf\_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW\_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`\_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf\_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI\_BGK\_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb[ 'dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b[ 'UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j ]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[ `7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa ]bXYX`h\UhWUfYZ ``hja Y`a UbU[ Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y ]XYbh]Z]YX`WUi gYg`cZd`U[ ]Uf]ga `]g`dccf`hja Y`a UbU[ Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'hja Y`Zcf fYgYUfW\žXfUZh]b[ `UbX`dfcdYf`fYZYfYbW]b[ `cZgci fWfg`]b`dfYdUf]b[ `U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb[ 'h\Y'9@&9'hi hcf]U`'UbX'ei ]n'k ]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhiHUg\_g

#### 5 WUXYa ]W = b Zcfa Uh cb ·

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb[ 'hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua ]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybhi fY/
- gh XYbhYei ]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

.

## ∍a U[Y'7fYX]h

Gc`c'DYfZcfa UbWY'A U\_]b['&\$%

#### 7F = 7CG

7F = 7CG Dfcj ]XYf 7cXY. \$\$\$-,;