

ARTS3270

FYZYW]b['cb'<]ghcf]Yg'UbX'<]ghcf]Ubg.'7 UdghcbY

HYfa 'Hk c''##'&\$%

7ci fgY⁻Cj Yfj]Yk

'GhJZZ'7cbhJWh'8YhJ]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	D\cbY
; fY[cfm9jcb	["Yj cb4 i bgk "YXi "Ui	6mUddc]bha Ybh	A 6 '&' '	-',)

GWXcc`7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc`C\\Z|\\\žA cfj\Yb'6fck\b'6i]\X]b[\\ž@Yj\Y``&\\z\&),

CdYb]b['<ci fg. 'A cbXUm! ': f]XUmž - Ua '!') da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . Ž*%'&'-',)', +\$)

9a U]`.'\<u>U`4 i bgk "YXi "Ui</u>

7ci fgY 8YW] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. <]ghcfm"

#b 'h]g 'Wldgrcby' Wci fgy' mci 'k]'``'c]b 'U``'h Y 'ch Yf'gh XYbhg'a U'cf]b[']b' <]grcfm]b' U'k YY_mgYa]bUf XYg][bYX 'rc 'dYfZyWiUbX' fYZYWicb 'h Y 'g_]`g' mci '\Uj Y 'YUfbYX']b 'mci f' \]grcfmXY[fYY"' #b'dUft]W\ 'Ufz'mci k]`` 'WcbXi Wimci f' ck b'a U'cffYgYUfW\ 'dfc YWh ZfUa]b['U'ei Yghjcb 'UVci hih\Y'dUghih\Uh\Wb'VY'UXXfYggYX \
\text{VmfYWcbghfi Who cZh YgY' Ybhg' Zfca 'df]a Ufmgci fW gzk \]'Y 'Yb[U[]b['k]h\ 'fYW bhgW.c`Ufg\]d']b 'mci f |
\bhYfdfYhUhcb cZh\YgY' Yj Ybhg'' < YfY'mci 'k]`` 'fYZYWicb 'h\Y' Z bXUa YbhU''df]b\yd' Yg 'cZh\Y' X]g\yd']b\z' \
\tau[Yh\Yf'k]h\ 'h\Y' gc\yU'' UbX'Yh]\wU'']a d`]\wUhcbg cZdUft]\w' `Uf' UddfcU\w\Yg' ic 'h\Y' 'dfUwhy\w' cZ <]grcfm' \
\f YUX]b[gzX]g\w gg]cb 'UbX'Z]\Y'Xk cf_ 'UfY' Zc\w gYX'cb 'XYj Y`cd]b['UddfY\yUhjcb cZh\Y' fc'Y'cZ\]grcf]\w'' \
_bck 'YX[Y']b'i bXYfghUbX]b['UbX']bZi Yb\yb['ci f' \wcbhYa dcfUfmk cf'X'' \Mci 'k]``YI d`cfY'h\Y'cf][]bgz \
\U[YbXUgz`UbX'a Yh\cXc`c[]Yg'VY\]bX'U'bi a VYf'cZ_Ym\wcbhYa dcfUfm\]grcf]\wU''UddfcU\wYgz`UbX'h\Y \
\wcbhfcj Yfg]\g'ih\Ym\Uj Y'gdUf_YX'k]h\]b' h\Y'X]g\yd']bY''' \Mci 'k]``\Wcbg]XYf'ih\Y'k Umg'\]grcf]\Ubg'UXUdhih\Y]f \
\dfcZ\gg]cbU''dfU\w]\w'g' ic 'di V']\wyb[U[Ya Ybhih\fci [\ a i gYi a gz'h\Y'YbhYffU]ba Ybh]bXi ghmz'UbX'h\Y \
\wchifng''' \Mci 'k]``U'gc'YI dYf]Yb\W' 'h\Y' Z bXUa YbhU''Hug_cZh\Y'\]grcf]\Ub'h\fci [\ mci f'ck b'fygYUf\w' dfc Y\w\ZUa]b['U'ei Ygh]cb'Uvci hih\Y'dUghih\Uh\\D' \Y' UXXfYggYX'\mfY\wchghfi \who fo'Zh\Yg\Y'y Ybhg''''

7ci fgY @YUfb]b['Ci hWca Yg

- $\% Ya d`cmUbX`Yj U`i UhY`a U`cf`df]bV]d`Yg`UbX`a Yh\cXc`c[]Yg`k]h\]b`h\Y`X]gW]d`]bY`cZ\]ghcfm```a Yh\cXc`c[]Yg`k]h\]b`h\Y`X]gW]d`]bY`cZ\]ghcfm```$
- ' "]XYbhjZma i `hjd`Y`Zcfa g`cZ\]ghcfm]b`h\Y`di V`]WXca U]bž'UbX`Yj U`i UhY`h\Ya `]b``][\hcZdfcZYgg]cbU` ghUbXUfXg`cZgW.c`Ufg\]d`UbX`Yh\]WU``VY\Uj]ci f
- ("XYg][b'UbX'WcbXi WhUb']bXYdYbXYbhfYgYUfWX dfc YWh
-) " fYZYWii dcb UbX]bhY [fUhY mci fick b Yl dYf]YbWyg Ugigh XYbhg cZ\]ahcfmUhl BGK

HYUW]b['GhfUhY[]Yg'

F UhjcbU'Y. 'H\YfY'UfY'bc ''YWi fYg']b'5F HG' &+\$"'H\Y'fc'Y'cZh\Y']bghi Wcf']g'tc'gmbh\Yg]gY'U'`Uf[Y'VcXmcZX]j YfgY'a UhYf]U'`Zcf'ghi XYbhgž'YUX]b['X]gW gg]cbg'cZ\]ghcf]WU'`a Yh\cXc`c[mUbX']hg'dc']h]WU'`UbX'gcW]U'` a d`]WUhjcbgž'UbX'Ugg]ghghi XYbhg']b'X]gWYfb]b['']b_U[Yg'VYhk YYb'h\Y'Wci fgY'a UhYf]U'g'UbX'h\Y]f'dfYj]ci g ghi X]Yg']b'\]ghcf]WU'`gi V'YWha UhhYf''''H\Y'gYa]bUf'Zcfa UhcV'][Yg'5@ghi XYbhg'hc'HU_Y'cb'Ub'Uwf]j Y]bhY'`YWi U'`fc'Y']b'UbU'mg]b['UbX'Wf]h]ei]b['h\Y'Wci fgY'fYUX]b[gžk\]W\UfY'a U]bm\]ghcf]c[fUd\]W\X\Wc'`UVcfUh]j Y'm]b'ga U'`'[fci dg'''5F HG' &+\$\Ug'hk c'Wca dcbYbhg.'f\%:5b'cb!WJa di g'a YYh]b[']b'U'hk c!\ci f'gYa]bUf'YUW\'k YY_'UWfcgg'h\Y'hYUW\]b['gYgg]cb/f\%:H\Y'Yei]j U'YbhcZUVci h+\\ci fg'cZ]bX]j]Xi U'`cf\Wc'`UVcfUh]j Y'k cf_'YUW\'k YY_"'H\]g']bWi XYg'U'a i gYi a 'j]g]hUbX'dfYdUfUhjcb'cZUb'YggUmUVci h]hz'UbX]bXYdYbXYbhfYgYUfW\']b'dfYdUfUhjcb'nc'k f]hY'U'fYgYUfW\'YggUmcb'U'ei Yghjcb'cf'dfcV'Ya 'cZh\Y'ghi XYbhfij ck b'XYg][b''

 $HYUWX]b[\ "UbX"@YUfb]b[\ "GlfUhY[\]Yg.\ "5g"h\]g"]g"U"gYa\]bUf"Wci\ fgYz"ci\ f"WcfY"hYUWX]b[\ "glfUhY[\ m]g"hc"k\ cf_k\]h\ mci\ "]bX]j\]Xi\ U"mUbX"]b"ga\ U""[\ fci\ dg'hc"X]gW gg"UddfcUWXYg"hc"\]ghcfmz"hc "fYZYWhcb"k \Uhmci \Uj Y$

ggYgga Ybh

ggYgga Ybh'HUg_g

WIND IN

>i gh]WY UbX Dc`]WY A i gYi a 'fk YY_YbXg'cb`mŁ

• 7]micZG\UXckg. \ bbYf'7]mi7f]a Y'/ 'A Um\Ya '% \ \ (,

BUhlcbu A Uflhla Y A i qYi a . T

- DUggYb[Yfg. H\Y'@cb['GYU'J cmU[Y'hc'U'BYk'@UbX
- H\Y'8Uf]b['G\]d.'H\Y'ghcfmicZ<A5G'JcmU[Yf"

5i ghfu`]Ub Ai gYi a . *

; Uff][UffUb[. `GYU'7 ci blfmfB 6. 'h\]g'Yl \]V]h]g'dUfhm\]ghcf]WU``UbX'dUfhm'Ubh\fcdc`c[]WU` \sharp 'gc 'ZcW g'cb h\Y'\]ghcf]WU``Y`Ya YbhgŁ

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh]g'gi Va]hhYX'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb q]a]'Uf]hmfYdcfhg"

5ggYgga Ybh' . FYgYUfVX '9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. %) \$\$ k cfXg

5XX]h]cbU`XYhJ]`g.

F Yei]fYa YbhgʻZcfʻh\YʻF YgYUfW\ '9ggUmk]```VY`X]gW ggYX`]bʻh\YʻK YY_'%GYa]bUfžk]h\`UXX]hJcbU`]bZcfa UhJcbʻdcghYXʻcbʻA ccXY"

 $\label{thm:limit} \begin{array}{ll} \mbox{Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\ \ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{array}$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi VYWfi]XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/ fYj]Yk ʻcZa Uf_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/