

LING5015

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'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	D\cbY
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7 fYX]h'Dc]bhg'*

Gi a a UfmcZh\Y 7ci fgY

H\]g'Wci fgY'XYj Y`cdg'mci f'fYgYUfW.'g_]``g'UbX'mci f'Uk UfYbYgg'cZ`Ub[i U[Y'i gY']b'gcW]Yhm''5```_Ym UddfcUWXYg'hc'hYl h'UbU`mg]g'k]```VY'fYj]Yk YX'hc'[]j Y'mci 'U'VfcUX'dYfgdYW]j Yžk \]W 'mci 'WUb'Udd`mhc'U fUb[Y'cZX]gW]d']bUfmWcbhYl hg']bWi X]b['Udd`]YX'`]b[i]gh]Wgž'ffUbg`Uh]cb'UbX']bHYfdfYhUh]cbža YX]U'UbX Wca a i b]WUh]cb'ghi X]Yg'UbX'bUffUh]j Y'UbU`mg]g'''Mci 'k]```XYj Y`cd'mci f'Wf]h]WU'h\]b_]b['UVci h\ck '`Ub[i U[Y i gY'fYZYWg'UbX'g\UdYg'ci f'k cfX'UbX'X]gW gg'gcW]U`'d\Ybca YbU'fY''[''Yei U']mz'gcW]U`'A gh]Wfz fYdfYgYbhUh]cb'cZUgmi a 'gYY_Yfgž'YhWL'h\fci [\'hYl h'UbU`mg]g'''Mci 'k]``'U`gc'`YUfb'UVci h\ck 'gdYU_Yfg a U_Y'Wkc]Wfg'hc'dYfZcfa 'h\Y]f']XYbh]hmz'Wc!cdYfUhY'UbX'Yl dfYgg'dc`]hYbYgg']b'Wcbj YfgUh]cbg'UbX'bUffUhY dUghYj Ybhg''' \$\frac{1}{2}\$ hYfa g'cZk f]hhYb'hYl hgz'mci 'k]``'Qh'XmVc\Yg]cb'UbX'Wc\YfYbW']b'UwXYa]Wk f]h]b['z'UbX h\]g'k]``\Y'd'mci 'VY'U'VYhhYf'k f]hYf''': cf'mci f'dfc YWgz'mci 'k]``'VY'ZfYY'hc'WxccgY'Zfca 'U'VfcUX'fUb[Y'cZhYI himdYg']bWi X]b['bYk gdUdYfgz'hfUbg`Uh]cb#bhYfdfYh]b['hYl hgz'UWXYa]WhYl hgz'bUhi fU``Wcbj YfgUh]cbg'UbX gcW]U''a YX]U'hYl hg''

7 ci fgY @YUfb]b[Ci hWca Yg

- %" ⇒ YbhjZmUbX``UVY``ZYUhi fYg`cZbUhi fU``Wcbj YfgUhjcbg`]b`9b[`]g\`i g]b[`Wcbj YfgUhjcb`UbU`mg]g hYVXb]ei Yq"
- &" F YV¢fX`UbX`lfUbqWf|VY`U`q\cfhV¢bj YfgUljcb`i g]b['lfUbgWf]dljcb W¢bj Ybljcbg"
- "8 Yg][b'U'fYgYUfW\'dfc'YWhig]b['X]gWcifgY'UbU'mg]g'a Yh\cXg"
- ("9jUiUhYh\Y'WcbbYWqcbg'VYkkYYb`Ub[iU[YigY'UbX'gcWjU'ž'dc`]njWU`'UbX'W`hifU`'ZcfaUhjcbg"
-) "8YjY`cd`Ub`UbU`mhjWJ``ghfUhY[mhc`Yld`cfY`dUhYfbg`cZgdc_Yb`cf`kf]hhYb`X]gWcifgY"
- *" 5f[iY'Zcf'h\Y'jU']X]micZU'X]gWcifgY'UbU'mh]WU''UddfcUW\'UbX']'i ghfUhY'h\Y'igY'cZh\Y'UddfcUW\ h\fci[\'Y|Uad'Yg"

HYUWX]b['GhfUhY[]Yg'

@YUfb]b['UbX'HYUW\]b['F Uh]cbU`Y

H\Y`WcbhYbhk Ug`WUfYZ``mgY`YWYX`hc`Wcj Yf`U``a U`cf`hfYbXg`]b`X]gWci fgY`UbU`mg]g"`<ck Yj Yfž'h\]g`a YUbg h\Unh\Y`j Uf]ci g`UddfcUW\Yg`UfY`bchYl d`cfYX`]b`XYdh\"`H\Y`U]a `]g`hc`[]j Y`Ub`]bhfcXi Wf]cb`hc`YUW\UddfcUW\`gc`h\Uhqh XYbhg`Wbb`qY`YWh\Y`VYqhUddfcUW\`Zcf`h\Y]f`fYqYUfW\`di fdcqYq"

HYUWX]b[GlfUlY[]Yg

7`UggYg'k]``fi b`Zcf'&`\ci fg`ZUW!hc!ZUW']b`U'k cf_g\cd`Zcfa Uh''H\Y'hYUW.]b['ghfUhY[m]g'hc'Wca V]bY'h\Y h\YcfYh]WU`[fci bX]b['k]h\'dfUWf]WU`Udd`]WUh]cbg"'A cghcZh\Y'h]a Y'k]``VY'U``cWUhYX'hc'[fci d'X]gW gg]cbg VUgYX'cb'h\Y'dfY!gYhX]gW gg]cb'ei Ygh]cbg'UbX'YI YfW]gYg'Ug'ci h]bYX']b'h\Y'Wci fgY'gWXYXi 'Y''@YWfi fY h]a Y'k]``VY``]a]hYX'Zcf'WUf]Z]WUh]cb'cZWcbWYdhg"'Ghi XYbhg'UfY'fYei]fYX'hc'dfYdUfY'h\Y]f'Ubgk Yfg'hc'h\Y YI YfW]gYg'UbX'UWf]j Y`mhU_Y'dUfh']b'WUgg'X]gW gg]cb": cf'h\Y'dfc'YWhUgg][ba Ybhghi XYbhg'k]``WUffmci h h\Y]f']bX]j]Xi U``ghi XmcZX]gWci fgY'UbU`mg]g'cb'gdc_Yb'cf'k f]hhYb'Zcfa g'cZX]gWci fgY'"

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8 YHJ]`g. `Ghi XYbhg`k cf_`]b`[fci dg`cZ' `hc`Ui X]c#]XYc!fYWcfX`U`' \$`a]bi hY`Wcbj YfgUh]cb`Ghi XYbhg`fYWV]j Y`U [fci d'a Uf_`Zcf`h\Y`fYWcfX]b[`fl%sı Ł"`9UW\`ghi XYbhHU_Yg`fYgdcbg]V]`]lm`Zcf`U`%\$!a]bi hY`gY[a YbhUbX gi Va]hg`h\Y`75`UbU`mg]g`cZh\UhgY[a Ybh]bX]j]Xi U``mf&\$ı Ł"`H\Y``Yb[h\`cZh\Y`hfUbgWf]dh]g`Uddfcl]a UhY`m %\$\$\$`k cfXg*cZUbU`mg]g"`Ghi XYbhg`fYWV]j Y`k f]hYb`ZYYXVUW_"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]h "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYY "hi fb]h "UbX"gh XYbhg"Xc "bchgYYh]h "UbX"gh XYbhg XYbhg"Xc "bchgYYh]h "UbX"Ybhg XYbhg XYb$

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8YHJ]`g. 'H\]g`]g`Ub`]bX]j]Xi U``HUg_"'Gh XYbhg`[]j Y`U`%\$`a]bi hY`dfYgYbhUh]cb`cb`h\Y]f`]bX]j]Xi U``8]gWci fgY 5bU`mg]g`Dfc`YWf`Gh XYbhg`fYWY]j Y`k f]hhYb`ZYYXVUW_`UbX`dYYf`ZYYXVUW_`]b`WUgg"

5XX]h]cbU`XYhU]`g.

DfYgYbhUhjcbg UfY fcghYfYX" Gh XYbhigi Va]hih\Y]f DD g`]XYg df]cf hc h\Y]f gVXYXi `YX dfYgYbhUhjcb"

5ggYgga Ybh' . Dfc YWh

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8 YHJ]`g. 'H\]g`]g`U`dfc^YWiUgg][ba YbhcZ' \$\$\$`k cfXg`YI Wi X]b[`fYZYfYbWig"`Ghi XYbhg`W\ccgY`Ub`UfYU`cZ]bhYfYgh]b`X]gWci fgY`ghi X]Yg`UbX`XYj Y`cd`U`dfc YWidfcdcgU`"`H\Y`dfcdcgYX`dfc`YWig\ci `X`U`gc`]bWi XY gUa d`Y`XUHU`UbU`mg]g`i g]b[`h\Y`W\cgYb`a Yh\cX`cZX]gWci fgY`UbU`mg]g"`Ghi XYbhg`fYWi]j Y`cfU``ZYYXVUW_Zfca `hi hcf`UbX`h\Y]f`dYYfg`fXi f]b[`dfYgYbHJh]cbgŁ`UbX`fYWi]j Y`k f]hhYb`ZYYXVUW_`VUgYX`cb`h\Y`k f]hhYb gi Va]gg]cb"

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- DU'lf]X[Yž'6"'-8]gWci fgY'UbU'mg]g. 5b']blfcXi Whjcb"'f&\$\&\L'di fW\UgY'Zfca 'I BGK 'Vcc_g\cd
- 6\UhjU'Yh'U`"'-5Xj UbWYg']b'8]gWti fgY'Gh X]Yg'f&\$\$, E!'Y6cc_']b'l BGK '@]VfUfm

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- 6cc_-HYI hz WcbhYI hz dfYhYI h' 7 f]h]WU`=ggi Yg`]b`8]gWci fgY`5bU`mg]g" f&\$\$(Ł
- 6cc_'-FY[]ghYfž'; YbfYž'UbX'GhmY'f&\$\$-Ł
- 6cc_'-'8]gWci fgY'cZWci fgY.'5b'cj Yfj]Yk 'cZfYgYUfWk']b'X]gWci fgY'ghi X]Yg'f&\$\$-Ł
- 6cc_'- *\text{blfcXi W}\text{cb hc 8}\text{gWci fgY Gh X}\text{Yg f&\$\$(\text{L})\$
- 6cc_'-'5ddfcUWXYg'hc'X]gWci fgY'f% (Ł

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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hY'Yd\cbY'9I hYfbU''Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\'Hi fb]r]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'r]\W_Yhbi a VYf'\Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfg\L'rc
]bWi XY'Ug'Yj]XYbW' 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']\WJh]cb'''=\maintention
UfY'' bUYY''mfYWY']j Y'U'r]\W_Yhbi a VYf\Zfvi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'rc'gdYW]\ZfwJ''mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYglmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i\\identity a Ubm Zcfa qžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiU\wbck iYX[Ya Ybh'

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7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU`
ghfi Whi fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa Uh]cb'k]h\ci h\UW_bck `YX[Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h\UhYX]hUbX'hc
d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k]h\ci h\Uddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI_BGK_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b['UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[``7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``ha Ybu[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\žXfUZn]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fW'g`]b`dfYdUf]b[`U```UggYgga Ybh']hYa g"

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7 ca d'Yhlb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhiHUg_g

5 WUXYa]W = b Zcfa Uh cb

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- fYei Yghg Zcf Yl hYbg]cb/
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- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
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- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

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7 cbj Ybcffg dYfgcbU d\chc"

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