

# **ARTS2212**

Gci h\YUgh'5g]U

HYfa 'H\fYY''##'&\$&\$

## 7ci fgY<sup>-</sup>Cj Yfj ]Yk

## GHJZ7cbHJWh8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
A]bUFcWg	9 0	WcbHJWha Y'j ]U Ya U]`	A cfj Yb <sup>·</sup> 6fck b ' ) -	

### GWXcc``7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b[ '<ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . 'Ž \* %'&'-' , ) ', +\$)

9a U]`. '<u>\U`4 i bgk "YXi "Ui</u>

## 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

#### Gi a a UfmcZh\Y 7ci fgY

H\Y'Wci fgY'ZcW gYg'cb'a cXYfb'Gci h\YUgh5g]U'g]bW'h\Y'YbX'cZh\Y'Wc`cb]U`'dYf]cXi bhj`h\Y'dfYgYbhXUm' \dghYUX'cZh\Y'hfUX]rbcbU``]bhYfdfYhUhjcbžk \]W\ gYYg'U'hfYbX'rck UfXg'Ui h\cf]hUf]Ub]ga ž'k Y'g\U``fYUX\]ghcfm'UbX'dc`]hJWg']b'WcbhYa dcfUfmGci h\YUgh5g]U'Ug'Ub'i bfYgc`j YX'WcbZ]WhVYk YYb'k c'Wca dYhJb[gmghYa g'!! X]WhUrcfg\]d'UbX'XYa cWfUWm''H\Y'Ui h\cf]hUf]Ub'fY[]a Yg'cZA UfWcgžGi \Uffcf]bWi X]b['th\Y'%\*') '\_]``[b[g']b'\deltaXcbYg]UtZA U\Uh\]fz@YY'? i Ub'MYk ž'h\Y'?\a Yf'F ci [Yž6i fa U#A mUba Ufzh\Y'a cbUfW\m UbX'a ]`]hUfm]b'H\U]`UbXz'UbX'J ]YhbUa 'g]bW'%', - 'k ]``'dfcj ]XY'gca Y'WUgY'ghi X]Yg'h\fci [\'k\]W\'rc UbU`mgY'Gci h\YUgh5g]Ub'dc`]hJW'`XmbUa ]Wgz']bWi X]b['Wcffi dh]cbz'bYdch]ga z'gcW]U`'a cj Ya Ybhgz'h\Y |a dUWhcZh\Y'\deltahY'\

#### 7 ci fgY @YUfb]b[ 'Ci hWca Yg

%" 5 bU`mgY`\_YmYj Ybhgž`]XYUgž'dYcd`Y`UbX`hfYbXg`Wcj YfYX`Vmh\Y`Wci fgY" &" A U\_Y`gW\c`Uf`mUf[i a Ybhg`i g]b[`]bXYdYbXYbhm`cWUhYX`UbX`WfYX]V`Y`Yj ]XYbW\'\ " 7 cbXi Wh`]bXYdYbXYbhgW\c`Uf`mfYgYUfW\

# 5ggYgga Ybh

 $\label{eq:hamiltonian} $$HY'a$ Uf_]b[ 'fi Vf]WZcf'h\Y'9ggUm\Ug'VYYb'i d`cUXYX']bhc'h\Y'A ccX'Y'g]hY'''$$ 

 $fYgYUfWX`dfc\Upsilon Wf'``Ghi XYbhg`g\ci``X`gi Va ]hiU`*\$\$`k cfX`UbU`mg]g`UbX`U`cbY`dU[Y`UbbchUhYX`V]V`]c[fUd\mbox{$h$.Y'Xi} Y`XUhY"$ 

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$ 

5ggYgga Ybh'& FYgYUfVX '9ggUm

GhUfh'XUhY. '&%#\$-#&\$&\$ '\$).\$\$ 'DA

**@Yb[h**. W \$\$\$ k cfXg

8 YW] g.

F YgYUfVW '9ggUmVfc\_Yb'Xck b']bhc—V]V`]c[fUd\mz'UVglfUWiUbX'Z]bU`YggUmGhi XYbhg'k f]hY'Ub'YggUmfWU" '\$\$\$'k cfXgŁ]b'fYgdcbgY'hc'cbY'cZh\Y'ei Ygh]cbg'i g]b['gWkc'Ufg\]d']b'Gci h\YUgh'5g]Ub'ghi X]Yg"': YYXVUW\_]g'h\fci [\']bX]j ]Xi U`'Wa a Ybhg'UbX'fi Vf]W'H\]g']g'h\Y'Z]bU`'UggYgga Ybh'Zcf'UhYbXUbW'di fdcgYg"

#### 5XX]h]cbU`'XYhU]`g.

5ggYgga Ybh' . F YgdcbgYg hc fYUX]b[g

GhUfhiXUhY.: fca '%( #\$- #&\$

@Yb[h\.')\$\$!\*\$\$'k cfXq

8YHJ]`g.

Ghi XYbhgʻgi Va ]hUʻfYgdcbgYʻfWU"') \$\$!\*\$\$ k cfXgŁˈncˈgYhei Ygh]cbgʻcbˈk YY\_`mfYUX]b[ gʻVYZcfYʻhk chi hcf]Uʻg"'': YYXVUW\_jg·Vm]bX]j ]Xi UʻWca a YbhgʻUbXʻ]b!WUggʻX]gW gg]cb"

#### 5XX]h]cbU`XYhJ]`g.

GYY'h\Y'Z]Y'"Hi hcf]U`FYUX]b[g'UbX'Ei Ygh]cbg'Zcf'8]gWigg]cb"]b'h\Y'A ccX`Y'g]hY'hc'Z]bX'h\Y'fY`Yj Ubh ei Ygh]cbg'Zcf'h\Y'ci fbU`g'Xi Y'Uh'hi hcf]U`g']b'k\YY\_g'&'UbX'("''FYUX]b[g'UfY'U``'X][]h]gYX'UbX'UWWigg]V`Y'hc ghi XYbhg'j ]U'h\Y'[fYYb'@Y[Ubhc'`]VfUfm`]b\_']b'h\Y'A ccX`Y'g]hY'Zcf'h\]g'Wci fgY"

•

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYH & fb & gYY & fb &$ 

5ggYgga Ybh (. HYgh

GhUfh:XUhY. "%\* #%#&\$&\$ "%&.\$\$ "DA

8YHJ]`g.

 $Ghi~XYbhg`Wca~d`YhY`U`hYghifWU"`\&`\ci~fgŁ`Ubgk~Yf]b[~ei~Ygh]cbg`VUgYX`cb`h\Y``YWhi~fYg"$ 

 $\label{thm:condition} \mbox{Hi fb]h]b"gYh]b" Label Label$ 

## 5 hm/b XUb VW F Yei ]fYa Ybhg

## 7ci fgY'GWXYXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

K YY_''-'Bcj Ya VYf'! % 'Bcj Ya VYf	@YWhifY	7 `Ugg'HYgh
	Hi hcf]U`	±bX]j ]Xi U``9I YfW]gY`8]gWigg]cbg`Wcbh]bi YX
		8 cW a YbrUfmcb 'h\Y 'H\U]'? ]b[ '
K YY_'%\$.'%* 'Bcj Ya VYf'! &\$'Bcj Ya VYf	@YWn fY	HYghi
	Hi hcf]U`	±bX]j ]Xi U``9I YfWjgY`8]gWigg]cbg'Wcbhjbi YX
	C b`]bY'5Wf]j ]hm	8 ]gW gg]cb 6cUfX UWfj ]hjYg Ug k Y`` Ug J ]Yk ]b[ 'UbX
		Wcaa Ybh]b[ˈcbːYUWkˈch\Yff@ˈ±bX]j ]Xi U`'9l YfW]gY
	I and the second	YI \ ]V]hg

## FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

 $H\YfY\UfY\bc\hYl\ Wcc\_g\fYei\ fYX\Zcf\h\g\Wci\ fgY$ "

### FYWca a YbXYX FYgci fWfg

 $D^YUgY^c[']bhc^hY^A ccX^YG]hY^Zcf^hY^Wci fgY^UbX^W]\underline{W}^cb^hY^Z]Y^{''}9ggUmE i Ygh]cbg^UbX^G]V^]c[fUd^]Yg'' k^]W^VghY^C[YUX]b[g^Zcf^YUW^hcd]W''$ 

.

## 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

G % % %

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## Hi fb]h]b Gi Va ]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhjb['hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [\'Hi fb]h]bz'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-', ) "''' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''=\mathred{Image: Wa Jhmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ub'Y! hYbg]cb\mathred{Image: Wa Jhmci h\Yf'f'Y'j UbhXcW a Ybhg\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k ]'`

JbWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hYbg]cb'Udd']\wUh]cb"'\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k ]'`

Ui hca Uh]\wU'`mfYW']j Y'U'h]\w\_Yhbi a VYf\mathred{Image: hY'Yd\cbY'mci 'k ]\tag{Y}\wu''mug\_'\mathred{Image: Wa'\mathred{Image: hY}\wu''mug\_'\mathred{Image: hY}\wu''mug\_'\mathred{Image: hY}\wu''mug\_'\mathred{Image: hY'Yd\cbY''mci 'k ]\tag{Y}\wu''mug\_'\mathred{Image: hY'\mathred{Image: hY'\mathred{I

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUhjj Y'gi Va ]gg]cb'XYHJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

# 5WUXYa]W±bZcfaUhjcb

: cf`YggYbh]U``gh`XYbh]bZcfa~Uh]cb`fY`Uh]b[~hc.

- fYei Yghg Zcf Yl hYbg]cb/`UhY gi Va ]gg]cbg [ i ]XY`]bYg/