

# **ARTS1630**

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# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

# GhJZZ 7 cbhJWh 8 YhJ] `g

## 7cbj Ybcfg

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## Hi hcfg

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M HJ_U'BU_U'}a U	m"bU_U1]a U4 i bgk "YXi "Ui	H65	A cfj Yb'6fck b <b>&amp;\$(</b>	
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### 7ci fgY 8YW] g

#### 7 fYX]h'Dc]bhg'\*

#### Gi a a UfmcZh\Y 7ci fgY

H\Y'Wci fgY']bfcXi WYg'h\Y'>UdUbYgY'gci bX'gmghYa žVUg]W[fYYh]b[g'Ug'k Y```Ug'U'bi a VYf'cZVUg]W >UdUbYgY'gfi Wi fYg'UbX'j cWJVi `UfmYI dfYggYX']b'<]fU[UbU'UbX'? UhJ\_UbU'gm``UVUf]Yg'h\UhUfY'YggYbh]U` hc `VUg]W>UdUbYgY'Wca a i b]WJh]cb"'H\]g'Wci fgY']g'Zcf'h\cgY'k\c'\Uj Y'bc'cf'U'a cghbc'\_bck 'YX[Y'cZ >UdUbYgY"'H\Y'Zc'`ck]b['Z]j Y'gcW]U'#W 'hi fU'hcd]Wg'k]``VY'Wcj YfYX.'%"; fYYh]b[g'UbX'a YYh]b['bYk'Zf]YbXg/&"'5g\_]b['UVci hh\]b[g'Ufci bX'mci /' "'8U]`mfci h]bYg/("'@cWJh]cbg/)"'@]\_Yg'UbX'X]g`]\_Yg"'C b'YUW\'hcd]Wzk\\]'Y'ghi Xm]b['h\Y'`Ub[i U[Yžmci 'UfY'W\U`Yb[YX'hc'X]gWcj Yf'X]ZZYfYbhUddfcUW\Yg'hc'j ]Yk]b['h\Y'k cf'X Ufci bX'mci '`]b[i ]gh]WJ'`mUbX'W 'hi fU``mž'UbX'UfY'[]j Yb'h\Y'cddcfhi b]hmhc'i bXYfghUbX'W ffYbhgcW]cW''hi fU` ]ggi Yg']b'>UdUb"'Nci 'UfY'U`gc']bhfcXi WYX'hc'`YUfb]b['fci h]bYg'bYW/ggUfmhc'gi WWYX']b'h\Y'ZJghdUW''YUfb]b['cZU'ZcfY][b'`UfY'"

H\]g`Wci fgY`]g`U`VY[]bbYfg!`Yj Y```Ub[i U[Y`Wci fgY"`5```ghi XYbhg`k\c`\Uj Y`\UX`gca Y`Yl dYf]YbWY`k]h\Th\Y`Ub[i U[Y`cf`h\fci [\`dfYj]ci g`]bghfi Wf]cbža i ghZ]```]b`h\Y`d`UWYa Ybhei Ygh]cbbU]fY`Uj U]`UV`Y`Uh\\hdg.#\U`"Ufhg"i bgk "YXi "Ui #ghi XYbhg#Wci fgYg#Ub[i U[Y!d`UWYa Ybhg"`Mci 'k]`` bchVY`dYfa ]hhYX`hc`Ybfc``]b`U``Ub[i U[Y`Wci fgY`h\Uh`]g`hcc`YUgmZcf`mci "`=Zh\]g`cVWV fgž`mci 'k]```VY fYei YghYX`hc`W\Ub[Y`mci f`Ybfc`a Ybh'

#### 7 ci fgY @YUfb]b[ Ci hWca Yg

%" +bffcXi WY cbYgY`ZVmi g]b[ 'g]a d`Y`>UdUbYgY"
&" Gi ghU]b g]a d`Y`Wcbj YfgUh]cbg']b >UdUbYgY`k \Yb'hU`\_]b[ 'UVci hcbYgY`ZUbX'cbY'g`XU]`mfci h]bYg"
' "FYUX`UbX`k f]hY`< ]fU[ UbUž? UhU\_UbU`Zi Ybhmž'UbX'fYWc[ b]gY'), '? Ub']"
("FY[ c[ b]gY`dfcV`Ya g`cZ]bhYfW`hi fU``]bhYfUWf]cb`k ]h\`ch\Yf`>UdUbYgY`gdYU\_Yfg"
) "K cf\_'Wc``UVcfUh]j Y`mk ]h\`ch\Yfg"

### HYUW ]b[ 'GhfUhY[ ]Yg'

 $\label{thm:continuous} H\Y'Wc fgY']g'Hui [\hh\fci [\k YY_m'YWh fYgž'hi hcf]U'g'UbX'gYa ]bUfg'''+b ``YWh fYgž'ghi XYbhg``YUb[i U[Y'UbX``Ub[i U[Y'YUfb]b[ž'ghUfh]b[Zfca 'VUg]WWcbWYdhg'''+b h\Y'WcbhYI hi U``_bck `YX[Y'cZ>UdUbYgY'`Ub[i U[Y'UbX``Ub[i U[Y'YUfb]b[ž'ghUfh]b[Zfca 'VUg]WWcbWYdhg'''+b h' hcf]U'g'UbX'gYa ]bUfgž'ghi XYbhg'dfUWf]WY'h\]Yf'>UdUbYgY'`Ub[i U[Y'g_]``g'Wc``YWh]j Y`mž'ghUfh]b['k]h\'VUg]WZbWh]cbg'UbX'k cf_]b['cb'Wca dfY\Ybg]j Y'HUg_g'''-bhYffY`Uh]b['k]h\'ch\Yf'Wci fgY'a Ya VYfg']b``YWh fYgž h' hcf]U'g'UbX'gYa ]bUfg']g'U'gc'YggYbhJU'''Ghi XYbhg'k]```VY'YI dYWhYX'hc'dUfhJ\W]dUhY']b'X]gW gg]cbgž'dU]fg'cf [fci d'HUg_g'h\fci [\ci hih\Y'Wci fgY'''5bmZcfa 'cZX]gWf]a ]bUhJcb'VUgYX'cb'Ub']bX]j ]Xi U'fg'U[Yž'Yh\b]W]hmž gYI i U`cf]YbHJhJcb'UbX'h\Y'`]_Y'k]```bchVY'hc'YfUhYX''$ 

±bHYfUWfjcb'HYghk]``VY`\Y`X`Xif]b['mcif'gYa]bUf"'Mcik]``'dYfZcfa''Ug\cfhfc`Y'd`Um]b'>UdUbYgY'k]h\'cbYcZmcif'WUggaUHYg'Zcf') 'a]biHYg"'AcfY'XYHU]`g'k]``'VY'UbbcibWX`]b'H\Y''YWffY'UbX'mcif'gYa]bUfWUgg"H\Y'dYfZcfaUbW'k]``'VY'fYWcfYX'UbX'aUf\_YX']bX]j]XiU``mVmH\Y'HYUWXYf"

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

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GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

: ]bU``9| Ua `k ]``VY`\Y`X`Xi f]b[ 'h\Y`i b]j Yfg]hmY| Ua `dYf]cX"`H\Y`Y| Ua `]g`Zcf`&`\ci fg"`H\Y`hYghdUdYf`k ]``VY a Uf\_YX`UZhYf`h\Y`Wca d`Yh]cb`cZY| Ua "

5XX]h]cbU`XYhU]`g.

 $H\Y`XYhJ]`g`cZh\Y`YIUa~k~]``VY`]bZcfa~YX`]b`h\Y``YWh~fY`k~YY\_\%$"$ 

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	GYa ]bUf	7 cbj YfgUhjcb dfUWhjWY UbX fc Y!d Um'
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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

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D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgHUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'ghi XYbh'Uhi BGK VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hž'I BGK 'Ghi XYbh'7cXY'cZ7cbXi Wh'UbX'5VWWdhUV`Y'I gY'cZ IBGK =7 H°F Ygci fW/g°Dc`]Wm