

# **ARTS3991**

5fhg·UbX·GcWJU`·GWJYbWYg·7UdghcbY

GYa YghYf'CbY''##'&\$%

## 7 ci fgY Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`q

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
DYHYf'G`YnU_	d"g`YnU_4 i bgk "YXi "Ui	*	F cca '' %) ž A cfj Yb 6fck b	-',)!&(&&

#### GW.cc`7cbhJWh =bZcfa Uhlcb

GWXcc`cZ<ia Ub]hYg'UbX'@Ub[iU[Yg

@cWUhjcb. 'GW\cc`'C ZZj\\xizA cfj Yb'6fck b'6i ]`X]b[ z'@Yj Y``&z\&),

CdYb]b[ < ci fq. A cbXUm!: f]XUmž-Ua !!(.() da

D\cbY. Ž\*%'&'-',) '%\*, %

: UI . Ž\*%'&'-', ) ', +\$)

9a U]`. \<u>U`4 i bgk "YXi "Ui</u>

#### 5 h Yb X Ub WY F Yei ] f Ya Yb h g

5 qh XYbh]q Yl dYWYX hc UhYbX U``WUqq WcbhUWi\ci fq Zcf U ZUW!hc! ZUW! fl & L'cf V YbXYX Wci fqY UbX WeadYhY'U``'UWAj ]hYq'Zef'U'VYbXYX'cf'Z ``micb`]bY'WeifqY"

5 gh XYbhk\c'Uff]j Ygʻa cfY h\Ub′%) a ]bi hYgʻ`UhYʻa UmVY dYbU`]gYX Zcf'bcb!UthYbXUbVW" Zgi VX U'dYbU`hm ]g^]a dcgYXz\*h\Y`ghi XYbha i ghVY`]bZcfa YX`j YfVU``mUhh\Y`YbX`cZWUgg^UbX`UXj ]gYX`]b`k f]h|b[ 'k ]h\]b`&( \ci fg"

ZUʻgh XYbhYl dYf]YbW'gʻ]`bYggža ]gUXj Ybh fYʻcfʻch\YfʻcVW ffYbW h\Uha U\_YgʻUVgYbW Zfca ʻU WUgg#JWNj jmi buj cjXUV`Yžcf`YI dYWmg`hc`VY`UVgYbhZfca`U`Zcfh\Wcajb[`WUgg#JWNj jmz`h\Ymg\ci`X`gYY\_ dYfa ]qq]cb Zfca 'h\Y'7 ci fqY'5i h\cf]|mz'UbX'k\YfY'Udd`]WUV`Yz'h\Y]f`fYei Yqhq\ci `X'VY'UW\can dUb]YX'VmUb cf][]bU`cf`WfflZYX`WcdmcZU'a YX]WU`WfflZWJHY`cf`ch\Yf`Zcfa 'cZUddfcdf]UHY`Yj ]XYbWY"

5 7 ci fqY 5 i h\cf|lma UmYl W qY U qhi XYbh2fca WUqqYq cf UWlij |h|Yq Zcf i d hc cbY a cbh\" < ck Y j Yfz h\Ym a UmUgg][b'UXX]hcbU`UbX#cf'U`hYfbUh]j Y'hUg\_g'hc'Ybgi fY'Wca d`]UbWY"'5'7 ci fgY'5i h\cf]hmWcbg]XYf]b['h\Y [fubhb[cZuvqYbWraighvYrguhqZ]YXrurgh XYbhk]```qh]```VYruv`Yrbc'a YYhrh\YrWcifqY'q``Yufb]b[cihWcaYq UbX#cfjciaYcZYUfb]b["5 gh XYbhgYY\_]b[ 'Uddfcj U`hc VY UVgYbhZcfa cfY h\Ub cbY a cbh\ a i ghUdd`m ]b'k f]hb['hc'h\Y'8YUb'UbX'dfcj]XY'U```cf][]bU``cf'WYfhZYX'gi ddcfhb['XcW a YbHJhcb"

cfacfy]bzcfaUt]cbUVcihthYUthYbXUbWYdfchcVc`g]bthYY:UW`tmcZ5fhgUbX'GcWJU` GWYbWrg. \htdg.#k k k "Uffig"i bak "YXi "Ui #W ffYbhah XYbha#UWJXYa \\mathbb{W}\delta Zcfa Uhcb#dfchcWc\a![i \]XY\\delta Ya#

5 WUXYa W = DZcfa Uhlcb

## 5ggYgga Ybh

#### 5ggYgga YbhHUg\_g

5ggYgga Ybh'tUg_	K Y][ \h	8i Y'8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
F YgYUfWX '9ggUm'	(\$1	&, #\$) #&\$% `\$(.\$\$`DA	%ž&
A ]X!HYfa 'HU_Y!\ca Y Ugg][ ba Ybh	&\$ı	\$-#\$(#&\$% `\$(`.\$\$`DA	(
GYa ]bUf`dfYgYbfUf]cb`k f]fY! i d`	&\$ı	&, #\$) #&\$% `\$(`.\$\$`DA	(
FYZYW¶jY'Ugg][baYbh	&\$I	&, #\$) #&\$% `\$(.\$\$`DA	&Ž

#### 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% F YgYUfW 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

**@Yb[h**\." <u>ž\$\$\$</u>"k cfXq

**8YHJ**]`g. `H\]g`]g`h\Y`Z]bU``UggYgga YbhZcf`UhYbXUbW'di fdcgYg"`&ž) \$\$`k cfXg"`Gh XYbhg`a i ghdfcXi W'`U Wf]h]WU`ž'UbU`mf]WU`ž'gW\c`Uf`mUWUXYa ]WYggUm'': YYXVUW\_j ]U`]bX]j ]Xi U``Wca a Ybhg"

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$ 

5ggYgga Ybh& A ]X!hYfa 'HU\_Y!\ca Y 'Ugg][ ba Ybh

GhUfhiXUhY. '&-#\$' #&\$% '%\$.\$\$'5A

@Yb[h\.'%) \$\$ 'k cfXg

 $8 YHJ]^g. \% $$^k cfXg''Gh XYbhg'a i ghWf]hJWJ^mUbU^mgY'Ugg][bYX'UfhJWYg'': YYXVUW_j JU']bXJJ JXi U^Mcaa Ybhg''$ 

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & gYH & gYY Hi & fb & gYY Hi & fb$ 

5ggYgga Ybh' . GYa ]bUf dfYgYbHJh]cb k f]hY!i d

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. %) \$\$ k cfXg

8YHJ]`g. '+\$\$'k cfXg"'7`Ugg'dfYgYbHJhJcbg'k]``'VY'Ugg][ bYX'H\fci [\ci hh\Y'gYa YgHYf"': c``ck]b['h\Y'dfYgYbHJhJcbz'gh XYbhg'gi Va]h'U'Wf]hJWJ``gi a a Ufm'UbX'fYZYWhJcb'cb'h\Y'Ugg][ bYX'UbX'WUgg'X]gW gg]cb" : YYXVUW\_j ]U']bX]j ]Xi U``Wca a Ybhg"

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b gYhh]b[. H\]g Ugg][ba Ybh]g gi Va]hhYX hAfci [\Thi fb]h]b UbX ghi XYbhg Wub gYY Hi fb]h]b g]a] Uf]hm fYdcfhg"$ 

5ggYgga Ybh'(. FYZYWhjj Y'Ugg][ba Ybh

GhUfhXUhY. Bch5dd`]WUV`Y

 $8YHU]^*g.^*, $$^k cfXg'''Gh XYbhg'a i ghUfh]W `UhY'h\Y'k Umg']b'k \]WX'h\Y]f^X]gW]d^]bUfm'hfU]b]b[ `Wci `X'VY'i gYX ]b'Z fh\Yf'gh Xmcf'h\Y'k cfX'cZk cf_'''$ 

 $\label{thm:continuous} \begin{tabular}{ll} Hi fb]h]b gYhh]b[. H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\Thi fb]h]b UbX gh XYbhg WUb gYY Hi fb]h]b g]a] Uf]hm fYdcfhg"$ 

#### Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gh XYbhg UfY Yl dYWYX hc di hh\Y]f bUa Yg UbX gh XYbhbi a VYfg cb Yj YfmdU[Y cZh\Y]f Ugg][ ba Ybhg"

#### Hi fb]h]b Gi Va ]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b[ 'rc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-', ) '''' %cf'Ya U]''h\Ya 'cb'<u>Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui"</u>"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'h]\W\_Yhbi a VYf\Zfca '9I hYfbU''Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY'Yj UbhXcW'a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WJh]cb'''\=\maintention
Uh\WJ'`mfYWY']j Y'U'h]\W\_Yhbi a VYf\ZVi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]\Z\WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU`mž'UggYgga YbhitUg\_g'a i ghiVY'gi Va ]lth/X'Y`YWfcb]WU`mj ]U'Y]th\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgtUbW'g'k \YfY'th\]g']g'bchdcgg]V`Yž']hk ]```VY'gtUhYX'cb'mci f'Wci fgY'g'A ccX`Y'g]hY'k ]th\U`hYfbUh]j Y'gi Va ]gg]cb'XYhU]`g''

#### @UhY 5ggYgga Ybh'DYbU`h]Yg

5b'UggYggYX'HUg\_']g'XYYa YX``UhY']Z]h']g'gi Va ]hhYX'UZhYf'h\Y'gdYW]Z]YX'h]a Y'UbX'XUhY'Ug'gYhci h']b'h\Y Wti fgY'@YUfb]b['A UbU[Ya YbhGmghYa 'f@A GŁ"

K cf\_'gi Va ]hhYX'Zci fhYYb'fl%( Ł'XUmg'UZhYf'h\Y'Xi Y'XUhY'k ]``VY'a Uf\_YX'UbX'ZYYXVUW\_'dfcj ]XYX'Vi hbc'a Uf\_ k ]``VY'fYWcfXYX"' ₹h\Y'k cf\_'k ci `X'\Uj Y'fYWr]j YX'U'dUgg'a Uf\_'Vi hZcf'h\Y``UhYbYgg'UbX'h\Y'k cf\_']g'U Wca di `gcfm'Wci fgY'Wca dcbYbhz'U'gh XYbh'k ]``VY'XYYa YX'hc'\Uj Y'a Yhih\UhfYei ]fYa Ybh''H\]g'XcYg'bch Udd`mhc'U'hUg\_'h\Uh']g'UggYggYX'Vi h'bc'a Uf\_']g'Uk UfXYX"

K cf\_'gi Va ]hh/X'hk YbhnlcbY'f&&XUmg'UZhYf'h\Y'Xi Y'XUhY'k ]``'bchVY'UWWdhYX'Zcf'a Uf\_]b[ 'cf'ZYYXVUW\_UbX'k ]``'fYWY']j Y'bc'a Uf\_'cf'[ fUXY"'=Zh\Y'UggYgga YbhHUg\_']g'U'Wca di `gcfmWca dcbYbhcZh\Y'Wci fgY'U ghi XYbhk ]``'Ui hca UhJWU``mZJ]`'h\Y'Wci fgY"

#### GdYWJU`7cbg]XYfUhJcb55dd`]WUhJcbg

 $\label{eq:linear_continuity} $$Mc_i = W_0 - W_$ 

G]W\_bYggža ]gUXj Ybhi fY'cf'ch\Yf'W]fW a ghUbWrg'VYmcbX'mci f'Wcbhfc``a Um

ł DfYj Ybhimci Zfca Wca d`Yhlb[ U.Wci fqY fYei ]fYa Ybhž

ł? YYd mci Zfca 'UthYbX]b[ 'Ub 'UggYggUV'Y 'UWnji ]tmž

ł Ghod moi gi Va ]hh]b[ 'UggYggUV'Y k cf\_ Zcf'U Wci fgYž

ł `G][ b]Z]\WbhmUZZY\Wimci f`dYfZcfa Ub\Wr`]b`UggYggUV`Y`k cf\_ž`VY`]h'U`Zcfa U``YbX!cZgYa YghYf`Yl Ua ]bUh]cbž`U`\WUgg`hYghž`U``UVcfUhcfmihYghž`U`gYa ]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa 'cZUggYgga Ybh'

#### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UVW]XYbhU`mWcdm]b[ 'Zfca `U`gci f\W' k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]uf]ga ]bhch\YZc``ck]b[ WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a ]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k ]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h'WfYX]h'

±DUddfcdf]UhY`dUfUd\fUg]b[.'W\Ub[]b['U'ZYk'kcfXg'UbX'd\fUgYg'k\]`Y`acghmfYhJ]b]b['h\Y`cf][]bU`ghfi WhifY`UbX`]bZcfaUh]cb`k]h\ci hUW\_bck`YX[YaYbh'H\]g'U`gc'Udd`]Yg`]b'dfYgYbhJh]cbg'k\YfY`gcaYcbYdUfUd\fUgYg`Ubch\Yf'g`]XYUg`cf'kcfXg'k]h\ci hWfYX]h''±iU`gc'Udd`]Yg`hc'd]YW]b['hc[Yh\Yf'ei chYg'UbXdUfUd\fUgYg`]bhc'U'bYk'k\c`Yžk]h\ci hfYZ/fYbW]b['UbX'U'ghi XYbhg'ck b'UbU`mg]g`hc'Vf]b['h\Y`aUhYf]U`hc[Yh\Yf''

7 c``i g]cb. 'k cf\_]b[ 'k ]h\ 'ch\ Yfg'Vi h'dUgg]b[ 'c\Z'h\ Y 'k cf\_'Ug'U'dYfgcb'g']b\X]j ]\Xi U``k cf\_"'7 c``i g]cb'U`gc ]b\Wi XYg'dfcj ]\X]b[ 'mci f'k cf\_'hc'Ubch\ Yf'ghi XYbh\ VYZcfY'h\ Y 'Xi Y 'XUhYzcf'Zcf'h\ Y' di fdcgY'cZh\ Ya d`U[ ]\Uf]g]b[ 'Uh\Ubmh]a Yz'dUm]b[ 'Ubch\ Yf'dYfgcb'hc'dYfZcfa 'Ub'U\WUXYa ]\Wh\Ug\_z'ghYU']b[ 'cf'U\\ello i ]f]b[ Ubch\ Yf'dYfgcb'g' U\WUXYa ]\Wk cf\_'Ub\X'\ello dm]b[ ']hz'c\ZYf]b[ 'hc'\\ello a d`YhY'Ubch\ Yf'dYfgcb'g' k cf\_'cf'gYY\_]b[ d\una Ybh\Zcf'\\ello a d`Yh\beta d`Y

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW\_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca k \]WX`\_bck`YX[Y`cZh\Ya`\Ug'VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ff`gY`Zld`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf\_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h`\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UqqYqqa Ybhcf\Wci fgY`UhI\_BGK\_`cf`Ubch\Yf`i b]\Yfq]\m"

7 cffYWhfYZYfYbWb[ dfUWhWyg.

• VYHYfa UbU[Ymcifh]a Y

# 7ci fgY GWX YXi `Y

# J]Yk 'WUgg'ha YhUV'Y

# H]a YHJV`Y

8 UhY	HmdY	7 cbhYbh
K YY_'%'&*': YVfi Ufm!'( A UfVX	GYa ]bUf	%"``Cf][]bg`cZKYghYfb`gW]YbWYZ`d\]`cgcd\mž DfYgcWfUh]Wg.`A]`Yg]Ubg
		? "F "`DcddYfž'6UW_`hc`h\Y`DfYgcWfUh]Wg
K YY_'&.') 'A UfW\'!'%% A UfW\	GYa ]bUf	&"``H\Y`hf]U``cZGcWfUhYg"`8]ggYbhž`Wf]h]WU``h\]b_]b[
		D`Uhc.'H\Y'5dc`c[micZGcWfUhYg
K YY_'' .'%&'A UfWX'!'% A UfWX	GYa ]bUf	' ":Di fcgYg:cZYXi WUh]cbž:di fY:UbX:Udd`]YX:ghi Xm
		="GWXYZZYf."FYZYWIjcbg"cb"9XiWUnjcbU``FY`YjUbWY""
K YY_'(.'%'A UfWX'!'&) A UfWX	GYa ]bUf	("``8Ya cWfUWmUbX`9Xi WUH]cbžWca a i b]hmcZ `YUfb]b[
		>c\b`8YkYm`8Ya cWfUWnV `9Xi WUhjcbž7\"`+
K YY_').'&*'A UfWX!'% 5df]`	GYa ]bUf	) "'H\Y': fYY'DfYggž]a dcftUbWY'cZX]ggYbh
Sulf		>"G"`A]``.`Cb'@ VYfhmcZH\ci[\hi/`8]gWigg]cb"
6fYU'&'5df]`'!', '5df]`		2 20 002
K YY_'*.'-'5df]`'!'%) '5df]`	GYa ]bUf	*"'GW]YbWYg'/ '\i a Ub]h]YgžgdYW]U`]nUh]cb"
		7 "D" 'Gbck . 'H\Y 'Hk c '7 i `hi fYg
K_YY_`+.`%*`5df]``!`&& 5df]`		

		B"'7\cag_m'H\ci[\hg'cb'±bhY``YWhU`g'UbX'h\Y GW\cc`g
K YY_'%%'%('A Um'!'&\$ A Um	GYa ]bUf	DcghA cXYfb]ga žgcWJU`Wcbghfi Whjj ]ga "'9Xi WUhjcb UbX'h\Y'GVJYbVW'K Ufg
		D"'G`YnU'GcWjc`c[mcZGWjYbWY/'9XiWUhjcb
K_YY_'%&.'&%'A_Um'!'&+ A_Um	GYa ]bUf	%&"`9XiWUh]cb`Zcf`]hg`ckb`gU_Y"
		7 UfX]bU``< YbfmBYk a Ub. `H\Y`≢XYU`cZU'I_b]j Yfg]lmi
K YY_'% .'&, 'A Um'!'' >i bY	GYa ]bUf	H\YfY`k]``VY`bc`gYa]bUf`h\]g`kYY_"

### FYgci fWyg

DfYgWf]VYX F Ygci fWYg

7 ci fgY fYUX]b[ g k ]``VY Uj U]V`Y cb`]bY ]b A ccX`Y"

FYWca a YbXYX FYgci fWYg

Bch'Uj U]`UV`Y

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

@f]YZmci h]bY`\ck 'gh' XYbhZYYXVUW\_flVch\ 'Zcfa U``UbX`]bZcfa U`Łcb'h\Y`Wci fgY`k]```VY`[ Uh\YfYXž\ck ']h k]``VY`UbU`mgYX`UbX`\ck ']hk]```VY`UbWYX`i dcb'hc']a dfcj Y`h\Y'ghi XYbh`YUfb]b[ 'YI dYf]YbW''': cf'YI Ua d`Yž mci 'a ][ \hX]gW gg`k \Uhk Ug`]XYbh]Z]YX`]b'dUghZYYXVUW\_UbX`\ck 'h\]g'Wci fgY`k Ug'W\Ub[ YX`hc'UXXfYgg`h\Y]ggi Y"Q

Gh XYbhZYXVUW\_k]``VY`h\fci [\`h\Y`Ubcbma ci g`cb`]bY`A m9I dYf]YbWY`gmghYa "

#### = U[Y'7fYX]h

CdYb`UVWYgg`Zfca`AYhfcdc`]hUb`AigYia`cZ5fhžBYk`Mcf\_ž\hhdg.#kkk"aYhaigYia"cf[#Ufh#Wc``YWNjcb#gYUfWX#('\*%\$)

7F = 7CG

7F = 7CG Dfc| ]XYf 7cXY. \$\$\$-,;