## MATHEMATICS ENRICHMENT CLUB. <sup>1</sup> Solution Sheet 17, September 17, 2013

1. There are 5 odd integer digits to choose from. Once one is chosen for the rst digit, only 4 remain, then 3. So there are 5 4 3 = 603 digit numbers with distinct odd digits.

2.

- 5. (a)  $a_6 = 6 + 5 + 4 + 3 + 2 = 20$ 
  - (b)  $a_n$  is simply the sum of integers from 2 ton, which is an arithmetic series, so  $a_n = \frac{n-1}{2}(n+2)$ .
  - (c)  $b_6 = 6^2 + 5^2 + 4^2 + 3^2 + 2^2 = 90$
  - (d) Some may recognize that h is the nth square pyramidal number (en. wi ki pedi a. org/wi ki /Square\_pyrami dal \_number) minus 1. The formula for thenth square pyramidal number is n/6 (n + 1)(2n + 1), so b<sub>h</sub> = n/6 (n + 1)(2n + 1) 1.
- (a) ABCB<sub>1</sub> is a parallelogram sinc BC is parallel to AB<sub>1</sub> and CB<sub>1</sub> is parallel to AB. Similarly CBC<sub>1</sub>A is a parallelogram. So now we know that is the midpoint of B<sub>1</sub>C<sub>1</sub>.

Now  $\B_1AC = \ACB$  because they are alternate. ID is the point at which the altitude from A meets BC then  $\DAC = 90$   $\ACD = 90$   $\ACB$  so  $\DAC + \B_1AC = 90$ , and AD is the perpendicular bisector  $oB_1C_1$ .

- (b) Since all the altitudes are also perpendicular bisectors of a triangle, and perpendicular bisectors of a triangle are concurrent, these altitudes are also.
- 7. P must be on the opposite side of the chor AB from O otherwise the angle with be zero. Instead, let the angle at P be , then the angle at O is 180 2. Setting these equal gives = 180=3=60.