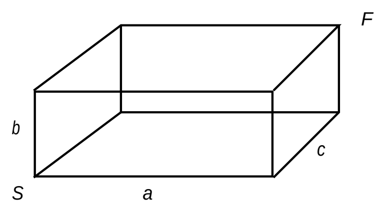
[redacted]

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 6, June 5, 2016

1. A spider, S is in one corner of a cuboid room, with dimensions $a \times b \times c$, and a y, F is in the opposite corner; see gure below. Find the shortest distance from S to F (Note that spiders can't y).



2. Working from left to right in a number, if the next digit is greater in value than the preceding digit, we say that the digits are strictly increasing; For example, 123, 247 and 367 are all 3-digit numbers with this property.

   Given a number has strictly increasing digits, what is the probability that it contains 5-digits?

3. How many ways are there to place one white king and one black king on an empty chessboard, such that they cannot attack each other?

4. Prove that $6^n + 8^n$ is divisible by 7 if and only if n is odd.

5. $ABCD$ is a parallelogram; X is the point on the diagonal BD. A line through X parallel to AB intersects AD at the point P; a line through X parallel to BC intersects AB at Q. Show that the area of the quadrilateral $APCQ$ is half the area of $ABCD$.

6. Let x be a real number. Denote by $[x]$ the largest integer less than or equal to x; For example, [ ] = 3. Find all positive real numbers $x; y$ satisfying the equation

$$[x][y] = x + y:$$

Senior Questions

1. In a chess tournament, every participant played with each other exactly once, receiving 1 point for a win, 1=2 for a draw and 0 for a loss. Is it possible that for every player P, the sum of points of the players who were beaten by P is greater than the sum of the points of the players who beat P?

2. At the end of the school year it became clear that for any arbitrarily chosen group of no less than 5 students, 80% of the marks \A" received by this group were given to no more than 20% of the students in the group. Prove that at lease 3=4 of all \A" marks were given to the same student.

3. Given the Fibonacci sequence $1; 1; 2; 3; 5; 8; 13; \ldots$ de ned by the second order recurrence relation, $F_{n+2} = F_n + F_{n+1}$