MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 2, May 14, 2018

1. What is the smallest number divisible by 1, 2, 3;:::, 10?

2. Suppose n is an integer greater than 1.

   (a) Show that $n^2 + n$ lies strictly between two squares and so cannot be a square.

   (b) Use the same idea to show that $n^4 + n^3 + n^2 + n$ is not a perfect square. (Note that it is when $n = 1$.)

3. Find the side length of the square whose base lies on the base of an isosceles triangle with sides 10, 10 and 12, with two vertices touching the equal sides 0f the triangle.

4. Find the largest positive integer that when divided into each of 364, 414 and 539 leave the same reminder.

5. Let AB be a chord of a circle centre O and let P be a point on its circumference. If $\backslash APB = \backslash AOB$, nd this angle.

6. There are 128 coins of two di erent weights, 64 of each. How can one always nd two di erent coins by performing no more than 7 weighings on a regular balance?

Senior Questions

1. It can be shown that the sum to in nity of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + :::= \frac{^2}{6}:$$

   Use this to nd the sum to in nity of the series

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + :::$$

   and

$$\frac{1}{1^2} \quad \frac{1}{2^2} + \frac{1}{3^2} \quad \frac{1}{4^2} + ::::$$

2. Robin Hood's apprentice still has a lot to learn when it comes to archery. She can always hit the target, but is equally likely to hit any point on the target. On a square target, what is the probability that she will land the arrow closer to the bullseye (the centre) than an edge?