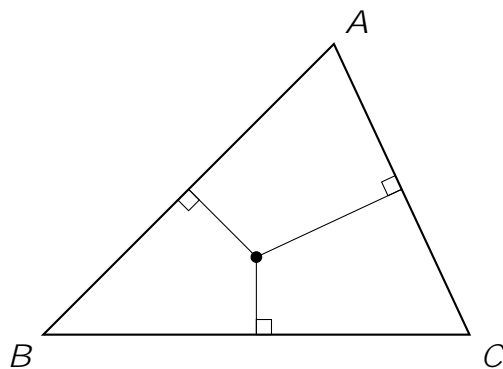


**MATHEMATICS ENRICHMENT CLUB.**  
**Problem Sheet 5, June 4, 2018**

1. If  $a$  and  $b$  are positive integers with  $a > b$ , and  $(a + b)^2 - (a - b)^2 > 29$ , find the smallest possible value of  $a$ .
2. If the straight line  $y = x + c$  meets the circle  $x^2 + y^2 = 1$  at a single point, find the value(s) of  $c$ .
3. Let  $ABC$  be a triangle. Prove that the perpendicular bisectors of the sides  $AB$ ,  $AC$  and  $BC$  intersect at a single point. (This point is called the circumcentre of the triangle.)



4. Without using a calculator, show that

$$\sqrt[3]{\frac{5}{13+18}} - \sqrt[3]{\frac{5}{13-18}} = 3:$$

Hint: Let  $x = a - b$  and then cube.

5. If  $x$  and  $y$  are positive integers which satisfy  $x^2 - 8x - 1001y^2 = 0$ , what is the smallest possible value of  $x + y$ ?  
 (AMC 2012 Senior Division Q23)

## Senior Questions

1. Suppose that  $g(x)$  is an odd function. Show that, if  $g$  is defined at  $x = 0$ , then  $g(0) = 0$ .
2. (a) Suppose that  $f(x)$  is an even function defined for all real  $x$  and differentiable throughout its domain. Show that  $f'(x)$  is an odd function.  
(b) Similarly, suppose that  $g(x)$  is an odd function defined for all real  $x$  and differentiable throughout its domain. Show  $g'(x)$  is an even function.